۱۹٫۲۵ آفرین

نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ...۱۶... کلاس ...دوازدهم سه شنبه...

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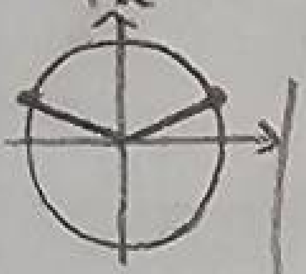
الف) $\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

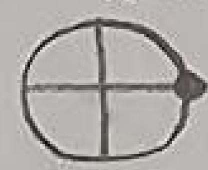
$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm \sqrt{25}}{4}$ ↗ $\frac{2}{4} = \frac{1}{2} = \sin x$ ↘ -2 ✗



$\boxed{\begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}}$ ✓

ب) $\underbrace{\cos 2x}_{2\cos^2 x - 1} + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0 \begin{cases} \cos x = 1 \\ \cos x = -\frac{3}{2} \ ✗ \end{cases}$



$\boxed{x = 2k\pi}$ ✓

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الف) $\underbrace{\sin^4 x - \cos^4 x}_{\underbrace{-\cos 2x}_{-\sin(\frac{\pi}{2} - 2x)}} = \sin 4x \Rightarrow \sin(2x - \frac{\pi}{2}) = \sin 4x$

$4x = 2k\pi + 2x - \frac{\pi}{2} \Rightarrow \boxed{x = k\pi - \frac{\pi}{4}}$ ✓

$4x = 2k\pi + \pi - 2x + \frac{\pi}{2}$

$\Rightarrow 6x = 2k\pi + \frac{3\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{3} + \frac{\pi}{4}}$ ✓

ب) $\underbrace{2\cos^2 x}_{\cancel{1} - \cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = \cancel{1} \Rightarrow \cos 2x = \underbrace{\sin 2x}_{\sin(\frac{\pi}{2} - 2x)}$

$2x = 2k\pi + \frac{\pi}{2} - 2x \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{8}}$ ✓

$2x = 2k\pi + \pi - \frac{\pi}{2} + 2x$ ✗

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow 2\cos^2 x + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0 \Rightarrow \begin{cases} \cos x = -1 \\ \cos x = \frac{1}{2} \end{cases} , \sin x \neq 0$

$\sin x = 0$ ✗

$x = 2k\pi \pm \frac{\pi}{3}$ ✓

ب) $2\sin^2 x - \sin x\cos x + \underbrace{\cos^2 x}_{1} = 2$

$\sin^2 x - \sin x\cos x + 1 = 2 \Rightarrow \underbrace{1 - \sin^2 x}_{\cos^2 x} = 2\sin\cos x$

$\cos x(\sin x + \cos x) = 0$

$\cos x = 0 \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}}$ ✓

$\sin x = -\cos x \Rightarrow \boxed{x = k\pi + \frac{3\pi}{4}}$ ✓

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الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \Rightarrow \frac{1}{2}[\cos(2x) + \cos(\frac{\pi}{2})]$

$= \frac{1}{4} \Rightarrow \cos 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow$

$\boxed{x = k\pi \pm \frac{\pi}{6}}$ ✓

ب) $\cos(2x - \frac{\pi}{9}) = \underbrace{-\sin 2x}_{\underbrace{\sin(-2x)}_{\cos(\frac{\pi}{2} + 2x)}} \Rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$\Rightarrow 2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x)$

$\cancel{2x} - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + \cancel{2x}$ ✗

$2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{7\pi}{18} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{\sin 4x}{\sin 2x} + 1$

$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$\frac{\sin 4x}{\sin 2x} + \cancel{1} - \cancel{\sin^2 x} = \cancel{1} - \cancel{\sin^2 x} \Rightarrow \frac{\sin 4x}{\sin 2x} = 0$

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$\sin 4x = 0 \Rightarrow x = \frac{k\pi}{4}$

$\sin 2x \neq 0 \Rightarrow x \neq \frac{k\pi}{2}$

$\boxed{x = \frac{2k\pi + \pi}{4}}$ ✓

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}\Rightarrow\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}\Rightarrow\left(\sin\frac{x}{2}\right)^2=\left(1+\cos\frac{x}{2}\right)^2\Rightarrow$$

$$\underbrace{\sin^2\frac{x}{2}}_{1-\cos^2\frac{x}{2}}=1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}\Rightarrow 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0\Rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0\Rightarrow\cos\frac{x}{2}=0$$

$$\cos\frac{x}{2}=-1$$

$$\frac{x}{2}=k\pi+\frac{\pi}{2}\Rightarrow x=\frac{k\pi}{2}+\frac{\pi}{4}$$

$$\frac{x}{2}=2k\pi+\pi\Rightarrow x=k\pi+\frac{\pi}{2}$$

$$\frac{5\pi}{4},\ \frac{3\pi}{2}\Rightarrow\left|\frac{3\pi}{2}-\frac{5\pi}{4}\right|=\boxed{\frac{\pi}{4}}$$

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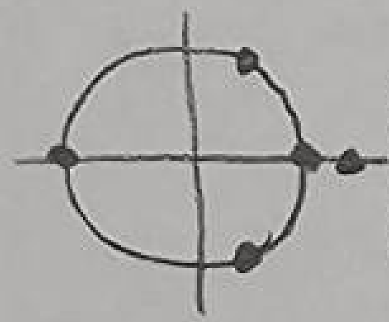
$$\cos^2x-\sin^2x\cos3x=1\Rightarrow\cos^2x-(1-\cos^2x)\cos3x=1\Rightarrow\cos^2x(1+\cos3x)-\cos3x=1$$ ۷

$$\cos^2x(1+\cancel{\cos3x})=1+\cancel{\cos3x}$$

$$1+\cos3x=0\Rightarrow3x=2k\pi+\pi\Rightarrow x=\frac{2k\pi}{3}+\frac{\pi}{3}$$

$$\cos^2x=1\Rightarrow x=k\pi$$



$\boxed{\text{۵ تا جواب}}$

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$$\frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)\Rightarrow\tan3x=\tan\left(\frac{\pi}{4}-x\right)\Rightarrow3x=\frac{\pi}{4}-x+k\pi$$ ۸

$$\Rightarrow\boxed{x=\frac{\pi}{16}+\frac{k\pi}{4}}$$

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$$\left(\sqrt{1+\sin2x}\right)^2=\left(\sqrt{2}\cos2x\right)^2\Rightarrow1+\sin2x=2\cos^22x\Rightarrow1+\sin2x=2(1-\sin^22x)$$

$$\Rightarrow1+\sin2x=2-2\sin^22x\Rightarrow2\sin^22x+\sin2x-1=0$$

$$\sin2x=1\Rightarrow2x=2k\pi+\frac{3\pi}{2}\Rightarrow x=k\pi+\frac{3\pi}{4}$$

$$\sin2x=\frac{1}{2}\Rightarrow2x=2k\pi+\frac{\pi}{6}\Rightarrow x=k\pi+\frac{\pi}{12}$$

$$2x=2k\pi+\frac{5\pi}{6}\Rightarrow x=k\pi+\frac{5\pi}{12}$$

جواب ۵ $=\frac{3\pi}{4}+\frac{\pi}{12}\rightarrow$ $\boxed{\text{دو جواب دارد}}$

برای $\frac{5\pi}{12}$ ق ق

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الف) $\sin^4x-\cos^4x=1\Rightarrow-\cos2x=1\Rightarrow\cos2x=-1\Rightarrow2x=2k\pi+\pi\Rightarrow x=k\pi+\frac{\pi}{2}$

مجموع جواب $=\frac{\pi}{2}+\frac{3\pi}{2}=\boxed{2\pi}$

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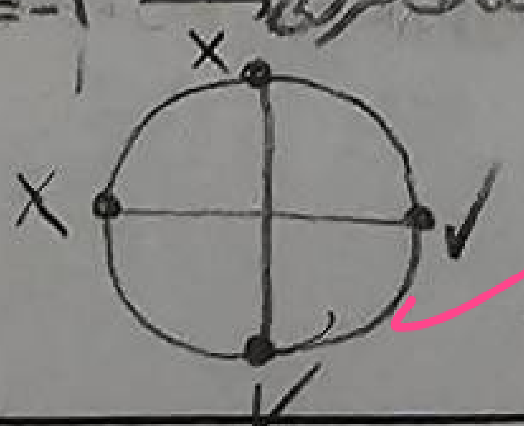
۱۰ ب) $\sin^3x-\cos^3x=-1\longrightarrow$ چک کردن نقاط مرزی

مجموعه جواب ها را می‌خواهد :)



$$\rightarrow0,\ \frac{3\pi}{2},\ 2\pi\Rightarrow0+\frac{3\pi}{2}+2\pi=\boxed{\frac{7\pi}{2}}$$

سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \tan\frac{x}{4}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \cot\frac{x}{4}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \rightarrow \tan\frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = 2k\pi + \pi$$

جواب‌ها

$k = 0 \rightarrow x = \pi$

$k = -1 \rightarrow x = -\pi$

$\longrightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$