«مطهره کمیلی» مطهره کمیلی تکلیف شماره‌ی ۱۴ مبحث مثلثات [illegible]

سوال ۱ ←

الف) $\cos 2x = 3\sin x - 1$

$\hookrightarrow \cos 2x \underset{-2\sin^2 x}{\textcircled{-1}} = 3\sin x - 2 \longrightarrow 0 = 2\overset{t^2}{\sin^2 x} + 3\overset{t}{\sin x} - 2 \longrightarrow \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm \sqrt{\overset{5}{9+16}}}{4} = \cancel{\frac{-8}{4}}$ غ ق ق یا $\boxed{\frac{1}{2}}$ ✓

$\sin x = \frac{1}{2} \longrightarrow x = \boxed{2k\pi + \frac{\pi}{6}}$ یا $2k\pi + \pi - \frac{\pi}{6}$ $\boxed{2k\pi + \frac{5\pi}{6}}$

ب) $\cos 2x + \cos x - 2 = 0 \longrightarrow \cos^2 x - \underbrace{\sin^2 x}_{-(1-\cos^2 x)} + \cos x - 2 = 0 \longrightarrow 2\cos^2 x + \cos x - 3 = 0$

$\hookrightarrow \cos^2 x - \sin^2 x$

جمع ضرایب $= 0$ ← ریشه‌ها 1 ، $\frac{c}{a}$

$\hookrightarrow \cos x \begin{cases} \to -\frac{3}{2} ✗ \\ \to 1 \longrightarrow \boxed{x = 2k\pi} \end{cases}$

سوال ۲ ←

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\hookrightarrow \underbrace{\cancel{(\sin^2 x + \cos^2 x)}}_{1} \underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = 2\sin 2x \cos 2x \longrightarrow -\cancel{\cos 2x} = 2\sin 2x \cancel{\cos 2x}$

$-\frac{1}{2} = \sin 2x \longrightarrow x = 2k\pi + \frac{7\pi}{6}$

$\hookrightarrow x = 2k\pi + \frac{11\pi}{6}$

ب) $2\cos^2 + 2\sin x \cos x = \cancel{1} \longrightarrow \cos 2x + \sin 2x = 0 \longrightarrow \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right) = 0 \longrightarrow$

$\hookrightarrow \cancel{1} + \cos 2x \qquad \hookrightarrow \sin 2x$

$\sin\left(2x + \frac{\pi}{4}\right) = \sin(\pi)$

$\Rightarrow 2x + \frac{\pi}{4} = \pi + 2k\pi \longrightarrow x = k\pi + \frac{3\pi}{8}$

$2x + \frac{\pi}{4} = 2k\pi + \pi - \pi \longrightarrow x = k\pi - \frac{\pi}{8}$

$\Big\} \Longrightarrow \boxed{x = k\pi - \frac{\pi}{8}}$

سوال ۳ ←

الف) $\cancel{\cot x}(2\cos x + 1) = \frac{1}{\sin x} \times \tan x \longrightarrow \left(2\cos x + 1 = \frac{1}{\cos x}\right) \times \cos x \longrightarrow 2\cos^2 x + \cos x - 1 = 0$

$\to \frac{-b \pm \sqrt{\Delta}}{2a} \to \frac{-1 \pm \sqrt{\overset{3}{1+8}}}{4} \begin{cases} \to -1 \longrightarrow \cos x = -1 \to x = \boxed{2k\pi + \pi} \\ \to \frac{1}{2} \longrightarrow \cos x = \frac{1}{2} \to x = \boxed{2k\pi \pm \frac{\pi}{3}} \end{cases}$

ب) $\underbrace{2\sin^2 x - \sin x \cos x + \cancel{\cos^2 x}}_{①} = 2 \longrightarrow \left(\sin^2 x - \frac{1}{2}\sin 2x = 1\right) \times 2 \to 1 - \cancel{\cos 2x} - \sin 2x - 2 = 0$

$\sin^2 x$

$\hookrightarrow \frac{1 - \cos 2x}{2} \qquad -\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$

$\longrightarrow \sin\left(2x + \frac{\pi}{4}\right) = \cancel{\frac{-1}{\sqrt{2}}}\ \underset{\sin\left(\frac{\pi}{4}\right)}{\frac{\sqrt{2}}{2}} \longrightarrow 2x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \longrightarrow \boxed{x = k\pi}$

$2x + \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{4} \longrightarrow \boxed{x = k\pi + \frac{\pi}{4}}$

$۴ = ۲k\pi + \pi - \frac{\pi}{۴} \rightarrow \boxed{x = k\pi + \frac{}{۴}}$

سوال ۴ ←

الف) $\cos(\frac{\pi}{۴}+x)\cos(\cancel{x-\frac{\pi}{۴}}) = \frac{1}{۴}$

$\hookrightarrow \sin(\frac{\pi}{۲}-(x-\frac{\pi}{۴})) \rightarrow \sin(\frac{۳\pi}{۴}-x) = \sin(\frac{\pi}{۴}+x)$

زاویه مکمل

$\rightarrow \cos(\frac{\pi}{۴}+x)\sin(\frac{\pi}{۴}+x) = \frac{1}{۴}$

$= ۲ \sin(۲x+\frac{\pi}{۲}) = \frac{1}{۴} \rightarrow \sin(۲x+\frac{\pi}{۲}) = \frac{1}{۲}$

$۲x+\frac{\pi}{۲} = ۲k\pi + \frac{\pi}{۶} \rightarrow \boxed{x = k\pi - \frac{\pi}{۶}}$

$۲x+\frac{\pi}{۲} = ۲k\pi + \pi - \frac{\pi}{۶} \rightarrow \boxed{x = k\pi - \frac{\pi}{۳}}$

ب) $\cos(۲x-\frac{\pi}{۹}) = -\sin ۴x \rightarrow \cancel{-}\sin(\frac{\pi}{۲}+\frac{\pi}{۹}-۲x) = \cancel{-}\sin(۴x)$

$\hookrightarrow \sin(\frac{\pi}{۲}-۲x+\frac{\pi}{۹})$

$-\frac{\pi}{۲} + \frac{\pi}{۹} + \cancel{۲x} + ۲k\pi + \pi = \cancel{۲x}$

↳ جواب ندارد

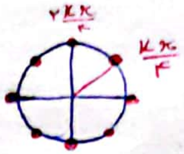
$\hookrightarrow \frac{\pi}{۲} + \frac{\pi}{۹} - ۲x = ۴x + ۲k\pi$

$\hookrightarrow \boxed{\frac{-۲k\pi}{۴} + \frac{۵\pi}{۳۶} = x}$

سوال ۵ ←

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \longrightarrow \frac{\sin 4x}{\sin 2x} + \cancel{1 - \sin^2 x} = \cos^2 x \longrightarrow \frac{\sin 4x}{\sin 2x} = 0$

$\boxed{k\pi \pm \frac{\pi}{4}}$ = جواب کلی ⟵ $\sin 2x \neq 0$ و $\sin 4x = 0$



سوال ۶ ←

$\frac{x}{2} = t \longrightarrow \frac{\sin t}{1+\cos t} = \frac{1+\cos t}{\sin t} \longrightarrow$ درصورتی عدد با معکوس خود برابر
± 1 = می‌شود که عدد
باشد

پس $\frac{\sin t}{1+\cos t} = \pm 1 \longrightarrow 1+\cos t = \sin t$
$\hookrightarrow 1+\cos t = -\sin t$

$\Longrightarrow 1 = (\sin t - \cos t)$
$1 = -(\sin t + \cos t)$

$\frac{\sqrt{2}}{2} \leftarrow 1 = \cancel{\sqrt{2}}\left(\sin\left(t - \frac{\pi}{4}\right)\right) \longrightarrow t = 2k\pi + \frac{\pi}{2}$ و یا $2k\pi + \pi$

$-\frac{\sqrt{2}}{2} \leftarrow -1 = \sqrt{2}\left(\sin\left(t + \frac{\pi}{4}\right)\right) \longrightarrow t = 2k\pi + \frac{3\pi}{2}$ و یا $2k\pi + \pi$

$\longrightarrow x = 2t \longrightarrow 2k\pi, 2k\pi + \pi$

تناقض $= |+\pi \mp \pi| = 2\pi$

سوال ۷ ←

$\cos^2 x - \sin^2 x \cos 3x = \cancel{1} \longrightarrow -\sin^2 x - \sin^2 x \cos 3x = 0$
$\hookrightarrow \cancel{1} - \sin^2 x$

$-\sin^2 x \,(1 + \cos 3x) = 0$

$\sin^2 x = 0 \longrightarrow 2k\pi, 2k\pi + \pi$

$1 + \cos 3x = 0 \longrightarrow x = \frac{[illegible]\pi}{3} + [illegible]$ $\longrightarrow$ ۴ جواب
(-1)

$\{\frac{\pi}{3}, [illegible] + k, 2\pi, 0\}$

سوال ۸ ←

$\frac{1 - \tan x}{1 + \tan x} = \tan 3x \longrightarrow x - \frac{\pi}{4} = 3x + k\pi \longrightarrow \boxed{x = -\frac{\pi}{8} + \frac{k\pi}{2}}$

$\downarrow \tan\left(x + \frac{\pi}{4}\right)$

سوال ۹ ←

$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x \xrightarrow{\text{به توان ۲}} 1 + \sin 2x = 2\cos^2 2x \longrightarrow \cancel{1} + \sin 2x - \cancel{2} + 2\sin^2 2x = 0$

$\hookrightarrow 1 - \sin^2 2x$ $\qquad -1$

جمع اولی و آخری ⟵ وسطی ⟵ ریشه ها ⟵

✓ $\frac{3\pi}{4}$ ⟵ $[0, \pi]$ در بازه‌ی $2k\pi + \frac{3\pi}{2}$ ⟵ $\sin 2x = -1$

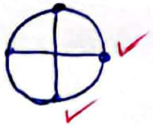
۳ جواب دارد ⟵ ✓ $\frac{5\pi}{12}$, $\frac{\pi}{12}$ ⟵ $[0, \pi]$ ⟵ $\sin 2x = +\frac{1}{2}$

سوال ۱۰ ←

$\sin^4 x - \cos^4 x = 1 \longrightarrow (\sin^2 x - \cos^2 x)(\cancel{\sin^2 x + \cos^2 x}) = 1 \longrightarrow \cos 2x = 1 \longrightarrow x = \begin{matrix} 0 \\ \pi \\ 2\pi \end{matrix} \longrightarrow$ ۳ جواب

$\downarrow \cos 2x \qquad \qquad 1$

$\sin^3 x - \cos^3 x \longrightarrow$



$\longrightarrow \begin{matrix} 0 \\ 2\pi \\ \frac{3}{2}\pi \end{matrix}$