عنوان

قسمت ب سوال 4 و 5 جابه جا نوشته شده است

تکلیف ۱۴ — ولید ازاعال

(۱

الف) $\cos 2x = 1 - 2\sin^2 x \Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x = 0$

$\sin x(2\sin x + 3) = 0$

① $\sin x = 0$ قق ② $\sin x = -\frac{3}{2}$ غق

$x = 2k\pi$ یا $x = 2k\pi + \pi$

ب) $2\cos 2x = 2\cos^2 x - 1 \Rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0 \quad 2\cos^2 x + \cos x - 3 = 0$

جمع ضرایب صفر است پس $\cos x = 1$ ، $\cos x = -\frac{3}{2}$

قق غق

$x = 2k\pi \pm 0 \Rightarrow x = 2k\pi$

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$\sin^4 x - \cos^4 x = \underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x \Rightarrow$

$-\cos 2x = \sin 4x \Rightarrow \cos(\pi - 2x) = \cos(\frac{\pi}{2} - 4x)$

$\pi - 2x = 2k\pi \pm (\frac{\pi}{2} - 4x) \Rightarrow$ ① $\pi - 2x = 2k\pi + \frac{\pi}{2} - 4x \Rightarrow x = k\pi - \frac{\pi}{4}$

② $\pi - 2x = 2k\pi - \frac{\pi}{2} + 4x \Rightarrow x = \frac{k\pi}{3} - \frac{\pi}{4}$

ب) $\underbrace{2\cos^2 x - 1}_{\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = 0 \Rightarrow \cos 2x + \sin 2x = 0 \Rightarrow \sin 2x = -\cos 2x$

$\cos(\frac{\pi}{2} - 2x) = \cos(\pi - x) \Rightarrow \frac{\pi}{2} - 2x = 2k\pi \pm (\pi - x) \Rightarrow$

$\frac{\pi}{2} - 2x = 2k\pi + \pi - x$ یا $\frac{\pi}{2} - 2x = 2k\pi - \pi + x$

① $x = 2k\pi + \frac{\pi}{2} \qquad x = \frac{2k\pi}{3} - \frac{\pi}{2}$

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \cos x(2\cos x + 1) = 1$

$\sin x \neq 0$

$2\cos^2 x + \cos x - 1 = 0 \Rightarrow$ $\cos x = -1$ ① ، $\cos x = \frac{1}{2}$ ②

$\boxed{c + a = b}$

① $x = 2k\pi + \pi$ غق زیرا به ازای این x ها $\sin x = 0$ می شود و $\cot x$ تعریف نشده!

② $x = 2k\pi \pm \frac{\pi}{3}$

ب) $\cos(3x - \frac{\pi}{9}) = -\sin 2x \Rightarrow \cos(3x - \frac{\pi}{9}) = \sin(-2x)$

$\cos(3x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} - (-2x)) \Rightarrow \cos(3x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$3x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x) \Rightarrow$ ① $3x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \rightarrow x = 2k\pi + \frac{\pi}{2} + \frac{\pi}{9}$

② $3x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \rightarrow 4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9}$

$x = \frac{k\pi}{2} - \frac{7\pi}{36}$

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$\cos(\frac{\pi}{4} - x) = \cos(x - \frac{\pi}{4})$ و $\frac{\pi}{4} - x + x + \frac{\pi}{4} = \frac{\pi}{2}$ پس این دو زاویه متمم اند

$\cos(\frac{\pi}{4} - x) = \sin(x + \frac{\pi}{4}) \Rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \Rightarrow$

$\frac{1}{2}\sin(2(x + \frac{\pi}{4})) = \frac{1}{4} \Rightarrow \sin(2x + \frac{\pi}{2}) = \frac{1}{2} \Rightarrow \cos 2x = \frac{1}{2}$

$2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow x = k\pi \pm \frac{\pi}{6}$

تکلیف ۱۶ — حسابان

ب) $2\sin^2x - \sin x\cos x + \cos^2x = 2 \Rightarrow \sin^2x + \sin^2x - \sin x\cos x + \cos^2x = 2$

$\sin^2x - \sin x\cos x = \frac{1}{\sin^2x+\cos^2x} \Rightarrow \cos^2x = -\sin x\cos x$

$\cos x \neq 0 \quad \cos x = -\sin x = \cos x = \sin(-x) \Rightarrow$

$\cos x = \cos(\frac{\pi}{2}+x) \Rightarrow$ ① $x = 2k\pi + \frac{\pi}{2} + x \Rightarrow \times$

② $x = 2k\pi - \frac{\pi}{2} - x \Rightarrow x = k\pi - \frac{\pi}{4}$

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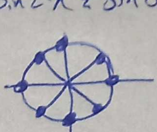
⑤ $\frac{\sin 4x + \sin 2x}{\sin 2x} = \underbrace{\sin^2x+\cos^2x}_{1}$ طرفین وسطین $\Rightarrow \sin 2x = \sin 2x + \sin 4x$

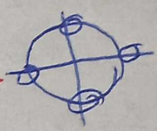
$\Rightarrow \sin 4x = 0$

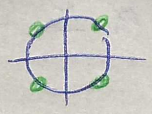
$\sin 4x = \sin 0$

① $4x = 2k\pi + 0$

② $4x = 2k\pi + \pi - 0$

$\Rightarrow x = \frac{k\pi}{2}$, $x = \frac{k\pi}{2} + \frac{\pi}{4}$ $\Leftarrow$ 

$\sin 2x \neq 0 \quad 2x \neq 2k\pi$, $2x \neq 2k\pi + \pi \Rightarrow$ ① $x \neq k\pi$ ② $x \neq k\pi + \frac{\pi}{2} \Rightarrow$ 

اجتماع جوابها $\Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$ 

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⑥ $\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \underbrace{\frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}}_{\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}}$ طرفین وسطین $\Rightarrow$

$\cos\frac{x}{2} \neq -1 \quad \sin\frac{x}{2} \neq 0$

$\sin^2\frac{x}{2} = (1+\cos\frac{x}{2})^2 \Rightarrow \sin^2\frac{x}{2} = 1 + \cos^2\frac{x}{2} + 2\cos\frac{x}{2} \Rightarrow$

$1 = \cos^2\frac{x}{2} + \sin^2\frac{x}{2}$

$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \xrightarrow{\div 2} \cos^2\frac{x}{2} + \cos\frac{x}{2} = 0 \Rightarrow \cos\frac{x}{2}(\cos\frac{x}{2}+1) = 0$

$\cos\frac{x}{2} = 0 \quad \cos\frac{x}{2} = -1$

غ‌ق‌ق زیرا مخرج کسر اول را صفر می‌کند و جز دامنه نیست

$\cos\frac{x}{2} = \cos\frac{\pi}{2} \Rightarrow \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \Rightarrow x = 4k\pi \pm \pi$

$x = 4k\pi + \pi$

$x = 4k\pi - \pi$

$x \in [-\pi, \frac{3\pi}{2}] \Rightarrow k = 0 \quad x = \pi, x = -\pi$

$k = 1 \quad x = -3\pi$ غ‌ق‌ق $\quad x = -5\pi$ غ‌ق‌ق

$|\pi - (-\pi)| = 2\pi$

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⑦ $\cos^2x - \sin^2x\cos 4x = \underbrace{\cos^2x + \sin^2x}_{1} \Rightarrow \sin^2x + \sin^2x\cos 4x = 0$

$\sin^2x(1+\cos 4x) = 0$

$\sin^2x = 0 \Rightarrow \sin x = 0$

$\cos 4x = -1$

۵ جواب

$x = 2k\pi + 0 \quad x = 0 \quad x = 2\pi$

$x = 2k\pi + \pi - 0 \quad x = \pi$

$4x = 2k\pi \pm \pi \quad x = \frac{\pi}{4}$

$x = 2k\pi + \frac{\pi}{4} \quad x = \frac{5\pi}{4}$

$x = 2k\pi - \frac{\pi}{4}$

کلاس ۱۶ — سوگند اردلان

(۸)

$$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \Longrightarrow \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}} = \frac{\cos x-\sin x}{\cos x+\sin x} = \frac{\sin 3x}{\cos 3x}$$

$\tan x \neq -1 \quad \cos x \neq 0 \quad \cos 4x \neq 0$

$$\cos x\sin 3x + \sin 3x\sin x = \cos 3x\cos x - \cos 3x\sin x \Longrightarrow \cos x\sin 3x + \cos 3x\sin x =$$
$$\cos 3x\cos x - \sin 3x\sin x \Longrightarrow \sin 4x = \cos 4x \xrightarrow[\text{طرفین تقسیم } \cos 4x]{} \tan 4x = 1$$

$$4x = k\pi + \frac{\pi}{4} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$$

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(۹)

$$\sqrt{1+\sin 2x} = \sqrt{1+2\sin x\cos x} = \sqrt{(\cos x+\sin x)^2} \to |\cos x+\sin x|$$

$|\cos x+\sin x| = \sqrt{2}\cos 2x \Rightarrow$ (1) $\underbrace{\cos x+\sin x}_{\sqrt{2}(\sin(x+\frac{\pi}{4}))} = \sqrt{2}\cos 2x$

$\sqrt{2}(\sin(x+\frac{\pi}{4})) = \sqrt{2}\cos 2x$

$\sin(x+\frac{\pi}{4}) = \cos 2x \Rightarrow \sin(x+\frac{\pi}{4}) = \sin(\frac{\pi}{2}-2x) \Rightarrow$ (1) $x+\frac{\pi}{4} = 2k\pi + \frac{\pi}{2} - 2x$

(1) $x = \frac{2k\pi}{3} + \frac{\pi}{12}$ $\quad x = \frac{\pi}{12} \quad x = \frac{9\pi}{12}$

(2) $x+\frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{2} + 2x$

(2) $x = -2k\pi + \frac{3\pi}{4} \quad x = \frac{3\pi}{4}$

(2) $\cos x+\sin x = -\sqrt{2}\cos 2x$

$\sqrt{2}\sin(x+\frac{\pi}{4}) = -\sqrt{2}\cos 2x \Rightarrow \sin(x+\frac{\pi}{4}) = \cos(\pi-2x) \Rightarrow$

$\sin(x+\frac{\pi}{4}) = \sin(\frac{\pi}{2}-\pi+2x) \Rightarrow \sin(x+\frac{\pi}{4}) = \sin(2x-\frac{\pi}{2})$

(1) $2x-\frac{\pi}{2} = 2k\pi + x + \frac{\pi}{4} \qquad x = 2k\pi + \frac{3\pi}{4} \qquad x = \frac{3\pi}{4}$

(2) $2x-\frac{\pi}{2} = 2k\pi + \pi - x - \frac{\pi}{4} \qquad x = \frac{2k\pi}{3} + \frac{5\pi}{12} \qquad x = \frac{5\pi}{12}$

[۴ جواب]

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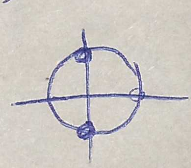
(۱۰)

الف) $(\underbrace{\sin^2x+\cos^2x}_{1})(\underbrace{\sin^2x-\cos^2x}_{-\cos 2x}) = 1 \Longrightarrow \cos 2x = -1$

$2x = 2k\pi + \pi \qquad 2x = 2k\pi - \pi$

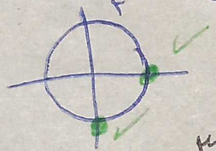
$x = k\pi + \frac{\pi}{2} \qquad x = k\pi - \frac{\pi}{2} \Rightarrow$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

ب) $\sin^3x - \cos^3x = -1$



$[x = 0 \quad x = 2k\pi \quad x = \frac{3\pi}{2}]$