عنوان

قسمت ب سوال 4 و 5 جابه جا نوشته شده است

۱۹ آفرین

ولید زارعیان تکلیف ۱۴

۱)

الف) $\cos 2x = 1-2\sin^2 x \Rightarrow 1-2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x = 0$

دقت کن $\sin x(2\sin x + 3) = 0$

$2\sin^2 x + 3\sin x - 2 = 0$ ۱/۵

① $\sin x = 0$ قق ② $\sin x = -\frac{3}{2}$ غ‌ق

$x = 2k\pi$ یا $x = 2k\pi + \pi$

$\sin x = \frac{1}{2} \to x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6}$ $(2\sin x - 1)(\sin x + 2) = 0$

ب) $\cos 2x = 2\cos^2 x - 1 \Rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0$ $2\cos^2 x + \cos x - 3 = 0$

جمع ضرایب صفر است پس $\cos x = 1$ ، $\cos x = -\frac{3}{2}$

قق غ‌ق

$x = 2k\pi \pm 0 \Rightarrow x = 2k\pi$

۲)

$\sin^4 x - \cos^4 x = \underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x \Rightarrow$

$-\cos 2x = \sin 4x \Rightarrow \cos(\pi - 2x) = \cos(\frac{\pi}{2} - 4x)$ ۲

$\pi - 2x = 2k\pi \pm (\frac{\pi}{2} - 4x) \Rightarrow$ ① $\pi - 2x = 2k\pi + \frac{\pi}{2} - 4x \Rightarrow x = k\pi - \frac{\pi}{4}$

② $\pi - 2x = 2k\pi - \frac{\pi}{2} + 4x \Rightarrow x = \frac{k\pi}{3} - \frac{\pi}{4}$

ب) $\underbrace{2\cos^2 x - 1}_{\cos 2x} + \underbrace{2\sin x\cos x}_{\sin 2x} = 0 \Rightarrow \cos 2x + \sin 2x = 0 \Rightarrow \sin 2x = -\cos 2x$

$\cos(\frac{\pi}{2} - 2x) = \cos(\pi - x) \Rightarrow \frac{\pi}{2} - 2x = 2k\pi \pm (\pi - x) \Rightarrow$

$\frac{\pi}{2} - 2x = 2k\pi + \pi - x$ یا $\frac{\pi}{2} - 2x = 2k\pi - \pi + x$

① $x = 2k\pi + \frac{\pi}{2}$ $x = \frac{2k\pi}{3} - \frac{\pi}{2}$

۳)

الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \Rightarrow \cos x(2\cos x + 1) = 1$ ۲

$\sin x \neq 0$

$2\cos^2 x + \cos x - 1 = 0 \Rightarrow$ ① $\cos x = -1$ ② $\cos x = \frac{1}{2}$

$\boxed{c + a = b}$

① $x = 2k\pi + \pi$ غ‌ق زیرا به ازای این x ها $\sin x = 0$ می‌شود

② $x = 2k\pi \pm \frac{\pi}{3}$

و $\cot x$ تعریف نشده!

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \Rightarrow \cos(2x - \frac{\pi}{9}) = \sin(-2x)$

$\cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} - (-2x)) \Rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi \pm (\frac{\pi}{2} + 2x) \Rightarrow$ ① $2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \longrightarrow x\,0 = 2k\pi + \frac{\pi}{2} + \frac{\pi}{9}$

ارفاق

② $2x - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \longrightarrow 4x = 2k\pi - \frac{\pi}{2} + \frac{\pi}{9}$

$x = \frac{k\pi}{2} - \frac{7\pi}{\cancel{36}}$ ۷۲

۴)

$\cos(\frac{\pi}{4} - x) = \cos(x - \frac{\pi}{4})$ و $\frac{\pi}{4} - x + x + \frac{\pi}{4} = \frac{\pi}{2}$ پس این دو زاویه متمم‌اند

$\cos(\frac{\pi}{4} - x) = \sin(x + \frac{\pi}{4}) \Rightarrow \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4} \Rightarrow$

$\frac{1}{2}\sin(2(x + \frac{\pi}{4})) = \frac{1}{4} \Rightarrow \sin(2x + \frac{\pi}{2}) = \frac{1}{2} \Rightarrow \cos 2x = \frac{1}{2}$

$2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow x = k\pi \pm \frac{\pi}{6}$

تکلیف ۱۶ — [illegible]

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \Rightarrow \sin^2 x + \sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\sin^2 x - \sin x\cos x = \underbrace{1}_{\sin^2 x + \cos^2 x} \Rightarrow \cos^2 x = -\sin x \cos x$

$\cos x \neq 0 \qquad \cos x = -\sin x = \cos x = \sin(-x) \Rightarrow$ ۲

$\cos x = \cos(\frac{\pi}{2} + x) \Rightarrow$ ① $x = 2k\pi + \frac{\pi}{2} + x \Rightarrow \times$

② $x = 2k\pi - \frac{\pi}{2} - x = x = k\pi - \frac{\pi}{4}$

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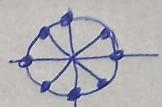
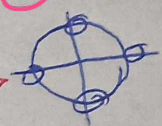
۵)

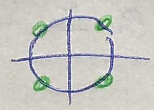
$\frac{\sin 4x + \sin 2x}{\sin 2x} = \underbrace{\sin^2 x + \cos^2 x}_{1} \xrightarrow{\text{طرفین وسطین}} \sin 2x = \sin 2x + \sin 4x$

$\Rightarrow \sin 4x = 0$

$\sin 4x = \sin 0$

① $4x = 2k\pi + 0$
② $4x = 2k\pi + \pi - 0$ $\Rightarrow$ $x = \frac{k\pi}{2}$, $x = \frac{k\pi}{2} + \frac{\pi}{4}$ $\Leftarrow$  ۲

$\sin 2x \neq 0$ $\quad 2x \neq 2k\pi$, $2x \neq 2k\pi + \pi \Rightarrow$ ① $x \neq k\pi$ ② $x \neq k\pi + \frac{\pi}{2}$ $\Rightarrow$ 

اجتماع جواب‌ها $\Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$ ✓ 

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۶)

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \underbrace{\frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}}_{\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}} \xrightarrow{\text{طرفین وسطین}}$

$\cos\frac{x}{2} \neq -1 \qquad \sin\frac{x}{2} \neq 0$

$\sin^2\frac{x}{2} = (1+\cos\frac{x}{2})^2 \Rightarrow \sin^2\frac{x}{2} = 1 + \cos^2\frac{x}{2} + 2\cos\frac{x}{2} \Rightarrow$

$1 = \cos^2\frac{x}{2} + \sin^2\frac{x}{2}$

$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \xrightarrow{\div 2} \cos^2\frac{x}{2} + \cos\frac{x}{2} = 0 \Rightarrow \cos\frac{x}{2}(\cos\frac{x}{2} + 1) = 0$ ۲

$\cos\frac{x}{2} = 0 \qquad \cos\frac{x}{2} = -1 \qquad \cos\frac{x}{2} = \cos\frac{\pi}{2} \Rightarrow$

غ‌ق‌ق زیرا مخرج کسر اولیه صفر شده و جز دامنه نیست

$\frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \Rightarrow x = 4k\pi \pm \pi$

$x = 4k\pi + \pi$

$x = 4k\pi - \pi$

$x \in [-\pi, \frac{3\pi}{2}] \Rightarrow \boxed{k = 0 \quad x = \pi, x = -\pi}$

$k = 1 \quad x = 3\pi$ غ‌ق‌ق $\quad x = 5\pi$ غ‌ق‌ق

$|\pi - (-\pi)| = 2\pi$ ✓

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۷)

$\cos^2 x - \sin^2 x\cos 3x = \underbrace{\cos^2 x + \sin^2 x}_{1} \Rightarrow \sin^2 x + \sin^2 x\cos 3x = 0$

$\sin^2 x(1 + \cos 3x) = 0$

۵ جواب

$\sin^2 x = 0 \Rightarrow \sin x = 0 \qquad \cos 3x = -1$

$x = 2k\pi + 0 \quad x = 0 \quad x = 2\pi \qquad 3x = 2k\pi \pm \pi \qquad x = \frac{\pi}{3}$

$x = 2k\pi + \pi - 0 \quad x = \pi \qquad x = 2k\pi + \frac{\pi}{3} \qquad x = \frac{5\pi}{3}$ ۲

$x = 2k\pi - \frac{\pi}{3}$ ✓

تکلیف ۱۴

(۱)

$$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} = \frac{\sin 3x}{\cos 3x} \Longrightarrow \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}} = \frac{\cos x-\sin x}{\cos x+\sin x} = \frac{\sin 3x}{\cos 3x}$$

$\tan x \neq -1 \quad \cos x \neq 0 \quad \cos 4x \neq 0$

$$\cos x \sin 3x + \sin 3x \sin x = \cos 3x \cos x - \cos 3x \sin x \Rightarrow \cos x \sin 3x + \cos 3x \sin x = \cos 3x \cos x - \sin 3x \sin x \Rightarrow \sin 4x = \cos 4x \xrightarrow[\text{طرفین تقسیم}\ \cos 4x]{} \tan 4x = 1$$

$$4x = k\pi + \frac{\pi}{4} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$$

۲

(۹)

$$\sqrt{1+\sin 2x} = \sqrt{1+2\sin x\cos x} = \sqrt{(\cos x+\sin x)^2} \rightarrow |\cos x+\sin x|$$

$$|\cos x+\sin x| = \sqrt{2}\cos 2x \Rightarrow ①\ \cos x+\sin x = \sqrt{2}\cos 2x$$

$$\sqrt{2}\left(\sin\left(x+\frac{\pi}{4}\right)\right) = \sqrt{2}\cos 2x$$

۱/۵

$$\sin\left(x+\frac{\pi}{4}\right) = \cos 2x \Rightarrow \sin\left(x+\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}-2x\right) \Rightarrow ①\ x+\frac{\pi}{4} = 2k\pi + \frac{\pi}{2} - 2x$$

$①\ x = \frac{2k\pi}{3} + \frac{\pi}{12} \qquad x = \frac{\pi}{12} \quad x = \frac{9\pi}{12} \qquad ②\ x+\frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{2} + 2x$

$②\ x = -2k\pi + \frac{3\pi}{4} \quad x = \frac{3\pi}{4} \qquad ②\ \cos x+\sin x = -\sqrt{2}\cos 2x$

$$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = -\sqrt{2}\cos 2x \Rightarrow \sin\left(x+\frac{\pi}{4}\right) = \cos(\pi-2x) \Rightarrow$$

$$\sin\left(x+\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}-\pi+2x\right) \Rightarrow \sin\left(x+\frac{\pi}{4}\right) = \sin\left(2x-\frac{\pi}{2}\right)$$

$①\ 2x-\frac{\pi}{2} = 2k\pi + x + \frac{\pi}{4} \qquad x = 2k\pi + \frac{3\pi}{4} \qquad x = \frac{3\pi}{4}$

$②\ 2x-\frac{\pi}{2} = 2k\pi + \pi - x - \frac{\pi}{4} \qquad x = \frac{2k\pi}{3} + \frac{5\pi}{12} \qquad x = \frac{5\pi}{12}$

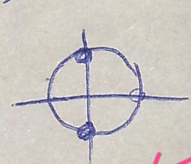
[۴ جواب] به ازای $x=\frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، این جواب را [illegible] است

(۱۰)

الف) $(\underbrace{\sin^2 x+\cos^2 x}_{1})(\underbrace{\sin^2 x-\cos^2 x}_{-\cos 2x}) = 1 \Longrightarrow \cos 2x = -1$

$2x = 2k\pi + \pi \qquad 2x = 2k\pi - \pi$

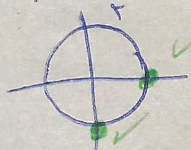
$x = k\pi + \frac{\pi}{2} \qquad x = k\pi - \frac{\pi}{2} \Big\} \Rightarrow$



$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

۲



ب) $\sin^3 x - \cos^3 x = -1$

$\left[\ x = 0 \qquad x = 2\pi \qquad x = \frac{3\pi}{2}\ \right]$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$$

یا $\sin 2x = -1$

یا $\sin 2x = +\frac{1}{2}$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x=\frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب رادیکال است نمی‌تواند

* معادله ۲ جواب دارد