۱۷, ۲۵ سازمانی

۱) الف) $2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow 0 = (2\sin x - 1)(\sin x + 2)$

$\sin x = -2$ غ ق ق

$\sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \pi - \frac{\pi}{6}$

$x = 2k\pi + \frac{\pi}{6}$

ب) $2\cos^2 x + \cos x - 1 = 0 \Rightarrow (2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ غ ق ق

$\cos x = -1 \Rightarrow x = 2k\pi \pm \pi$

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۲) الف) $\sin^2 x - \cos^2 x = -\cos 2x = \sin 2x \Rightarrow \cos 2x = \cos\left(\frac{\pi}{2} + 2x\right)$

$\Rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x$

$2x = 2k\pi - \frac{\pi}{2} - 2x$

$\Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{12}$

ب) $1 + \cos 2x + \sin 2x = 1 \Rightarrow \cos 2x = -\sin 2x \Rightarrow \cos 2x = \cos\left(\frac{\pi}{2} + 2x\right)$

$\Rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x$ یا $2x = 2k\pi - \frac{\pi}{2} - 2x$

$\Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

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۳) الف) $2\cos^2 x + \cos x - 1 = 0 \quad (2\cos x - 1)(\cos x + 1) = 0$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi$ ق ق ق

$\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

$\sin x \neq 0 \Rightarrow x \neq k\pi$

ب) $\cos^2 x - \sin x \cos x = 2\cos^2 x \Rightarrow -\sin x = \cos x \Rightarrow \cos x = \cos\left(\frac{\pi}{2} - x\right)$

$x = 2k\pi + \frac{\pi}{2} - x \Rightarrow x = k\pi + \frac{\pi}{4}$

$x = 2k\pi - \frac{\pi}{2} + x$

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۴) الف) $\cos\left(\frac{\pi}{2} + 2x\right)\sin\left(\frac{\pi}{2} + x\right) = \frac{1}{4} \Rightarrow \sin\left(\frac{\pi}{2} + 2x\right) = \frac{1}{4}$ ۱,۲۵

$\Rightarrow \cos 2x = \frac{1}{4} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$

$x = k\pi \pm \frac{\pi}{6}$

ب) $\cos\left(2x - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6} + 2x\right)$

$2x - \frac{\pi}{6} = 2k\pi + \frac{\pi}{6} + 2x$

$2x - \frac{\pi}{6} = 2k\pi - \frac{\pi}{6} - 2x \Rightarrow x = \frac{k\pi}{2}$

۵) (۱٫۵)

$$\frac{\sin 4x+\sin 2x}{\sin 2x}=1 \Rightarrow \sin 2x=\sin 4x+\sin 2x \Rightarrow \sin 4x=0$$

$$4x=k\pi \Rightarrow x=\frac{k\pi}{4}$$

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۶) (۱٫۲۵)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos x}{\sin x} \Rightarrow 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}=\underbrace{\sin^2\frac{x}{2}}_{1-\cos^2\frac{x}{2}}$$

$$\Rightarrow 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \Rightarrow \cos\frac{x}{2}=0 \Rightarrow \frac{x}{2}=k\pi+\frac{\pi}{2} \Rightarrow x=2k\pi+\pi$$

$$\cos\frac{x}{2}=-1 \quad (k=-1 \Rightarrow -\pi,\ k=0 \Rightarrow \pi)$$

$$\frac{x}{2}=2k\pi+\pi \Rightarrow x=4k\pi+2\pi$$

$$\pi-|\pi|=0$$

<این بازه جواب ندارد>

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۷) (۱٫۱۵)

$$2-\sin^2 x-\sin^2 x\cos 2x=2 \Rightarrow \sin^2 x+\sin^2 x\cos 2x=0 \Rightarrow \sin^2 x(1+\cos 2x)=0$$

$$\sin^2 x=0 \Rightarrow 2x=k\pi \Rightarrow x=\frac{k\pi}{2}$$

$$\cos 2x=-1 \Rightarrow 2x=2k\pi+\pi \Rightarrow x=k\pi+\frac{\pi}{2}$$

$k=0 \Rightarrow \frac{\pi}{2}$, $k=1 \Rightarrow \pi$, $k=2 \Rightarrow \frac{3\pi}{2}$, $k=3 \Rightarrow 2\pi$, $k=4 \Rightarrow 0$

۴ جواب (به جواب‌های مشترک توجه کنید) ← ۵ جواب

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۸) (۱)

$$\tan 2x+\tan x\tan 2x=1-\tan x \Rightarrow \tan 2x+\tan x=1-\tan x\tan 2x \xrightarrow{\div 1-\tan x\tan 2x}$$

$$\tan 2x=\frac{\tan x+\tan 2x}{1-\tan x\tan 2x}$$

$$\tan 2x=1 \qquad 2x=k\pi+\frac{\pi}{4} \Rightarrow x=\frac{k\pi}{2}+\frac{\pi}{8}$$

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۹) (۱٫۷۵)

$$1+\sin 2x=2\cos^2 2x \Rightarrow \sin 2x=\cos 4x \Rightarrow \cos\left(\frac{\pi}{2}+2x\right)=\cos 4x$$

$k=0 \Rightarrow \frac{\pi}{6}$, $k=1 \Rightarrow \alpha$

$$4x=2k\pi+\frac{\pi}{2}+2x \Rightarrow x=k\pi+\frac{\pi}{4}$$

$$4x=2k\pi-\frac{\pi}{2}-2x \Rightarrow x=\frac{k\pi}{3}-\frac{\pi}{12}$$

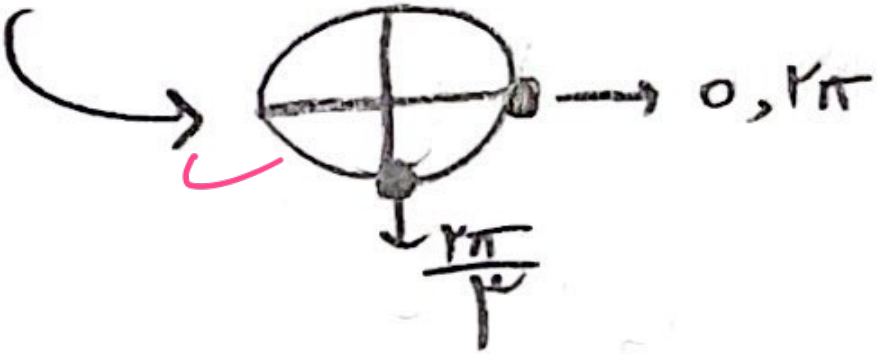
$k=0 \Rightarrow -\frac{\pi}{12}$ ✗, $k=1 \Rightarrow \frac{\pi}{4}$ ✓, $k=2 \Rightarrow \frac{7\pi}{12}$ ✓

(۱۵)

الف) $\sin^2 x - \cos^2 x = 1 \Rightarrow \cos 2x = -1$

$2x = 2k\pi + \pi \Rightarrow x = k\pi + \frac{\pi}{2}$

ب) امتحان می‌کنم!



سوال ۴ قسمت ب

$\cos(2x - \frac{11\pi}{6}) = -\sin 2x$

$\cos(2x - \frac{11\pi}{6}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{11\pi}{6} = 2k\pi \pm (\frac{\pi}{2} + 2x)$

حالت (+) ↗ $2k\pi + \frac{11\pi}{6} + \frac{\pi}{2} = 0$ ✗

حالت (−) ↘ $2x - \frac{11\pi}{6} = 2k\pi - \frac{\pi}{2} - 2x$

$4x = 2k\pi - \frac{7\pi}{12} \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{48}$

سوال ۵

$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x$ (1)

$\frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$

$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

سوال ۶

$\frac{\sin \frac{x}{2}}{1 + \cos \frac{x}{2}} = \tan \frac{x}{4}$

$\frac{1}{\sin \frac{x}{2}} + \cot \frac{x}{2} = \frac{1 + \cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1}{\tan \frac{x}{4}} = \cot \frac{x}{4}$

$\tan \frac{x}{4} = \cot \frac{x}{2} \rightarrow \tan \frac{x}{4} = \tan(\frac{\pi}{2} - \frac{x}{2})$

$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{2} \rightarrow \frac{x}{4} < k\pi + \frac{\pi}{2} \rightarrow x < 2k\pi + \pi$

جواب‌ها ↗ $k = 0 \rightarrow x = \pi$

↘ $k < -1 \rightarrow x < -\pi$ → اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۷

$\cos^2 x - \sin^2 x \cos 4x = 1$ $\xrightarrow{\cos^2 x = 1 - \sin^2 x}$

$1 - \sin^2 x - \sin^2 x \cos 4x = 1 \rightarrow \sin^2 x (1 + \cos 4x) = 0$

- $\sin x = 0$
- $\cos 4x = -1$

$\sin x = 0 \rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$ (۳، ۲، ۱)

$\cos 4x = -1 \rightarrow 4x = 2k\pi + \pi \rightarrow x = \frac{2k\pi + \pi}{3} \rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (۵، ۴)

معادله ۵ جواب دارد

سوال ۹

$\sqrt{1 + \sin 4x} = \sqrt{2} \cos 2x$ $\xrightarrow{\text{توان ۲}}$ $1 + \sin 4x = 2\cos^2 2x$

$1 + \sin 4x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0$

- $\sin 2x = -1$
- $\sin 2x = \frac{1}{2}$

$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2}$ $\xrightarrow[k=1]{0 \le x \le \pi}$ $x = \frac{3\pi}{4}$ ✓

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6}$ $\xrightarrow[k=0]{0 \le x \le \pi}$ $x = \frac{\pi}{12}$ یا $\frac{5\pi}{12}$ ✓ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید کرده

به ازای $x = \frac{5\pi}{12}$ عبارت $\cos 2x$ منفی می‌شود پس جواب را نمی‌پذیرد

★ معادله ۲ جواب دارد