پارمیس بنی‌زاده   دوازدهم دختر B   تکلیف شماره ۱۴

۱۹ آفرین

۱) الف) $\cos 2x = 3\sin x - 1 \rightarrow 3\sin x = \cos 2x + 1$

$3\sin x = 2\cos^2 x = 2(1-\sin^2 x)$ ۲

$2\sin^2 x + 3\sin x - 2 = 0 \qquad \sin x = \frac{-3 \pm 5}{4} = \frac{1}{2}, -2$ غ ق ق

$\sin x = \frac{1}{2} \qquad x = 2k\pi + \frac{\pi}{6}, \ 2k\pi + \frac{5\pi}{6}$

ب) $\cos^2 x - \sin^2 x + \cos x - 2(\cos^2 x + \sin^2 x) = 0$

$-\cos^2 x - 3\sin^2 x + \cos x = 0 \Rightarrow -\cos^2 x - 3(1-\cos^2) + \cos$

$2\cos^2 x + \cos x - 3 = 0 \Rightarrow \cos x = 1, \ \frac{-3}{2}$ غ ق ق

$\cos x = 1 \qquad x = 2k\pi$

۲) الف) $\overset{1}{\cancel{(\sin^2 x + \cos^2 x)}}(\sin^2 x - \cos^2 x) = 2\sin 2x \cos 2x$

$2\sin 2x \cos 2x = -\cos 2x \quad \begin{cases} \cos 2x = 0 \\ 2\sin 2x = -1 \end{cases}$ ۲

$\cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2}$

$\sin 2x = \frac{-1}{2} \qquad 2x = \left\{2k\pi - \frac{\pi}{6}, \ 2k\pi - \frac{5\pi}{6}\right\}$

$x = \frac{k\pi}{2} + \frac{\pi}{4}$ یا $x = \left\{k\pi - \frac{\pi}{12}, \ k\pi - \frac{5\pi}{12}\right\}$

$x = \left\{k\pi \pm \frac{\pi}{12}\right.$ یا $\left.\frac{k\pi}{2} + \frac{\pi}{4}\right\}$

ب) $2\sin x \cos x = 1 - 2\cos^2 x = \sin^2 x - \cos^2 x$

$\sin 2x = -\cos 2x$

$2x = k\pi + \frac{3\pi}{4} \Rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{8}$

۲) الف

$$\frac{\cos x}{\sin x} \times (2\cos x + 1) = \frac{1}{\sin x}$$

$$\Rightarrow 2\cos^2 x + \cos x = 1 \Rightarrow \cos x = 1 - 2\cos^2 x$$

۱,۷۵

$$\cos x = \sin^2 x - \cos^2 x = -\cos 2x$$

$$\cos 2x = -\cos x = \cos(\pi - x)$$

$$2x = 2k\pi \pm (\pi - x)$$

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده می‌شود

پس جواب غیر قابل قبول محسوب می‌شود

$$\begin{cases} 2x = 2k\pi + \pi - x \to x = \frac{2}{3}k\pi + \frac{\pi}{3} \\ 2x = 2k\pi - \pi + x \to x = 2k\pi - \pi \end{cases}$$

ب)

$$2\sin^2 x - \sin x \cos x = 2 - \cos^2 x$$

$$\cancel{2\sin^2 x} - \sin x \cos x = \cancel{2\sin^2 x} + \cos^2 x$$

$$\begin{cases} \cos x = 0 \to x = k\pi + \frac{\pi}{2} \\ \cos x = -\sin x \to x = k\pi - \frac{\pi}{4} \end{cases}$$

۴) الف

$$\cos\left(\frac{\pi}{4} + x\right)\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2} \times \left(\cos 2x + \cos\frac{\pi}{2}\right) = \frac{1}{4}$$

$$\Rightarrow \cos 2x + 0 = \frac{1}{2}$$

$$2x = 2k\pi \pm \frac{\pi}{3} \qquad x = k\pi \pm \frac{\pi}{6}$$

۲

ب

$$-\sin 2x = \cos\left(2x + \frac{\pi}{2}\right)$$

$$\Rightarrow \cos\left(2x - \frac{\pi}{9}\right) = \cos\left(2x + \frac{\pi}{2}\right)$$

$$\cancel{2x} - \frac{\pi}{9} = 2k\pi + \cancel{2x} + \frac{\pi}{2} \to$$ جواب ندارد

$$2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \Rightarrow 4x = 2k\pi - \frac{7\pi}{18}$$

$$x = \frac{k\pi}{2} - \frac{7\pi}{72}$$

Year Month Date ( ) Name :

$$\frac{\sin 4x + \sin 2x}{\sin 2x} = \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} \quad (5$$

$$\frac{\cancel{\sin 2x}\,(2\cos 2x + 1)}{\cancel{\sin 2x}} = \sin^2 x + \cos^2 x = 1 \quad 2$$

$$\Rightarrow 2\cos 2x = 0 \qquad 2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} \times \frac{1-\cos \frac{x}{2}}{1-\cos \frac{x}{2}} = \frac{\sin \frac{x}{2} \times (1-\cos \frac{x}{2})}{1-\cos^2 \frac{x}{2}} \quad (6$$

$$\frac{\cancel{\sin \frac{x}{2}} \times (1-\cos \frac{x}{2})}{\sin^2 \frac{x}{2}} = \frac{1-\cos \frac{x}{2}}{\sin \frac{x}{2}} \quad 2$$

$$\frac{1}{\sin \frac{x}{2}} + \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1+\cos \frac{x}{2}}{\sin \frac{x}{2}}$$

$$\frac{1-\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1+\cos \frac{x}{2}}{\sin \frac{x}{2}}$$

$$1 - \cos \frac{x}{2} = 1 + \cos \frac{x}{2} \Rightarrow 2\cos \frac{x}{2} = 0 \qquad x = 2k\pi + \pi$$

مجموعه جواب $= \{-\pi , \pi\}$ $\qquad |\pi - (-\pi)| = 2\pi$

$$-\sin^2 x \cos 3x = 1 - \cos^2 x = \sin^2 x \quad (7$$

$$\begin{cases} \sin^2 x = 0 & x = k\pi \\ \cos 3x = -1 & 3x = 2k\pi + \pi \qquad x = \frac{2}{3}k\pi + \frac{\pi}{3} \end{cases} \quad 2$$

$[0, 2\pi] \longrightarrow \{0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$ ۵ جواب

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right) \quad (8$$

2

$$\tan 3x = \tan\left(\frac{\pi}{4} - x\right)$$

$$3x = k\pi + \frac{\pi}{4} - x \Rightarrow 4x = k\pi + \frac{\pi}{4}$$

$$x = \frac{k\pi}{4} + \frac{\pi}{16}$$

Date ( ) Name :

$\sqrt{1+2\sin x\cos x} = \sqrt{2}\cos 2x \longrightarrow \cos 2x > 0$ (۹

توان ۲ $\longrightarrow 1+\sin 2x = 2\cos^2 2x = 2\left(\frac{1+\cos 4x}{2}\right)$

$\cancel{1}+\sin 2x = \cancel{1}+\cos 4x$

$\cos 4x = \sin 2x \quad , \quad \cos 4x = \sin\left(\frac{\pi}{2}-4x\right)$

$\sin 2x = \sin\left(\frac{\pi}{2}-4x\right)$

۲

$$\begin{cases} 2x = 2k\pi + \frac{\pi}{2} - 4x & x = \frac{k\pi}{3}+\frac{\pi}{12} \\ 2x = 2k\pi + \pi - \frac{\pi}{2} + 4x & x = -k\pi - \frac{\pi}{4} \end{cases}$$

مجموعه جواب $= \left\{\frac{\pi}{12} , \frac{5\pi}{12} , \frac{9\pi}{12}\right\}$ معادله ۲ جواب دارد

غ ق ق زیرا $\cos\frac{5\pi}{6}$ مقدار منفی دارد

$\sin^4 x - \cos^4 x = (\cancel{\sin^2 x+\cos^2 x})(\sin^2 x-\cos^2 x) = 1$ (۱۰

1

$\Rightarrow -\cos 2x = 1 \quad \Rightarrow \cos 2x = -1$

۱/۲۵

$2x = 2k\pi + \pi \quad x = k\pi + \frac{\pi}{2}$

مجموعه جواب $= \left\{\frac{\pi}{2} , \frac{3\pi}{2}\right\}$

ب) $\sin^3 x - \cos^3 x = (\sin x-\cos x)(1+\sin x\cos x)$

$$\underbrace{\sqrt{2}\times\sin\left(x-\frac{\pi}{4}\right)}_{R=(-\sqrt{2},\sqrt{2})} \times \underbrace{(1+\sin x\cos x)}_{=1+\frac{1}{2}\sin 2x} = -1$$

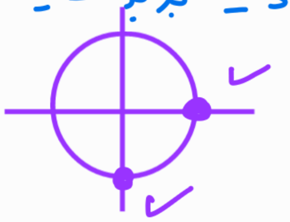
$R = \left\{\frac{1}{2} , \frac{3}{2}\right\}$

حداقل مقدار عبارت $= -\sqrt{2}\times\frac{1}{2} = \frac{-\sqrt{2}}{2}$

جواب های این معادله همواره بزرگتر از ۱- هستند $\Leftarrow$ معادله جواب ندارد

در سوالات به فرم $\pm \sin^n x \pm \cos^n x$ برابر 1 یا -1 . فقط کافی است نقاط اصلی را چک کنی !

$\sin^3 x - \cos^3 x = 1$



$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$