تکلیف شماره ۱۹ — دوازدهم دختر B — مرتضی ریاضی

الف) $\cos 2x = 3\sin x - 1 \longrightarrow 1 - 2\sin^2 x = 3\sin x - 1$

$2\sin^2 x + 3\sin x - 2 = 0$ $\qquad \sin^2 x + 3\sin x - 4 = 0$

$(\sin x + \frac{4}{2})(\sin x - \frac{1}{2}) = 0$

$\sin x = \frac{1}{2}$ ✓

$\sin x = -2$ غقق

$\sin x = \frac{1}{2}$

$x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0$ $\qquad 2\cos^2 x + \cos x - 2 ~~\cos~~ - 1 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$ $\qquad \xrightarrow{a+b+c=0}$

$\cos x = 1$ ✓

$\cos x = \frac{c}{a} = \frac{-3}{2}$ α غقق

$\cos x = 1$ $\qquad x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = \sin 4x$ $\qquad -\cos 2x = \sin 4x$

$(\cancel{\sin^2 x + \cos^2 x})(\sin^2 x - \cos^2 x) = -\cos 2x$

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$-\cos 2x = 2\sin 2x \cos 2x$

$\rightarrow \cos 2x = 0$ $\qquad 2x = 2k\pi \pm \frac{\pi}{2}$ $\qquad x = k\pi \pm \frac{\pi}{4}$

$\rightarrow 2\sin 2x = -1$ $\qquad \sin 2x = \frac{-1}{2}$

$\rightarrow 2x = 2k\pi - \frac{\pi}{6}$ $\qquad x = k\pi - \frac{\pi}{12}$

$\rightarrow 2x = 2k\pi - \frac{5\pi}{6}$ $\qquad x = k\pi - \frac{5\pi}{12}$

ب) $2\cos^2 x + 2\sin x\cos x = 1$ $\qquad 2\cos^2 x - 1 = -2\sin x\cos x \Rightarrow$ $\qquad \cos 2x = -\sin 2x$

$2x = k\pi + \frac{\pi}{4}$ $\qquad x = \frac{k\pi}{2} + \frac{\pi}{8}$

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الف) $\cot x\,(2\cos x + 1) = \frac{1}{\sin x}$ $\qquad \cos x(2\cos x + 1) = 1$ $\qquad a + c = b$

$2\cos^2 x + \cos x = 1$ $\qquad 2\cos^2 x + \cos x - 1 = 0$

$\rightarrow \cos x = -1$

$\rightarrow \cos x = \frac{-c}{a} = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3}$, $\cos x = -1 \rightarrow x = 2k\pi + \pi \rightarrow$ غیرقابل قبول $\rightarrow$ ریشه مخرج را صفر می‌کند.

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\sin^2 x - \sin x\cos x = 1 \xrightarrow{\div \cos^2 x} \tan^2 x - \frac{\sin x\cos x}{\cos x \cdot \cos x} = \frac{1}{\cos^2 x}$

$\tan^2 x - \tan x = \tan^2 x + 1$ $\qquad \tan x = -1$ $\qquad x = k\pi - \frac{\pi}{4}$

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$\cos(\frac{\pi}{4} + x)(\cos(x - \frac{\pi}{4})) = \frac{1}{4}$ $\qquad \cos(\frac{\pi}{4} + x)\overset{\sin}{\cancel{\cos}}(\frac{\pi}{4} - x) = \frac{1}{4}$ $\qquad \sin(2x + \frac{\pi}{2}) = \frac{1}{2}$ $\qquad \cos 2x = \frac{1}{2}$

$2x = 2k\pi \pm \frac{\pi}{2}$ $\qquad x = k\pi \pm \frac{\pi}{4}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x = \cos(\frac{\pi}{2} + 2x) \longrightarrow 2x - \frac{\pi}{9} = -2x - \frac{\pi}{2} + 2k\pi$ $\qquad x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x$ $\qquad \sin 4x + \sin 2x = \sin 2x$ *

$4x = k\pi$ $\qquad x = \frac{k\pi}{4}$

$\rightarrow$ $\sin 2x \neq 0$ *

$x \neq \frac{k\pi}{2}$

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$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$, $\frac{1-\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\frac{1-\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$ $\quad 1+\cos\frac{x}{2} = 1-\cos\frac{x}{2}$ $\quad 2\cos\frac{x}{2} = 0$ $\quad \cos\frac{x}{2} = 0$

$\frac{x}{2} = 2k\pi \quad x = 4k\pi$

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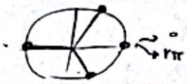
$\cos^2 x - \sin^2 x\cos 3x = 1 \quad \cos^2 x - 1 = \sin^2 x\cos 3x$ $\quad \sin^2 x = 0$

$-\cancel{\sin^2 x} = \cancel{\sin^2 x}\cos 3x \to \cos 3x = -1$

$\cos 3x = -1 \quad 3x = 2k\pi+\pi \quad x = \frac{2k\pi}{3}+\frac{\pi}{3}$, $\sin x = 0 \quad x = k\pi$

[illegible]جواب



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$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \quad \sqrt{(\cos x+\sin x)^2} = \sqrt{2}(\cos^2 x - \sin^2 x) = \sqrt{2}(\cos x+\sin x)(\cos x-\sin x)$

$|\cos x + \sin x| = \sqrt{2}(\cos x+\sin x)(\cos x-\sin x) \quad \cos x - \sin x = \pm\frac{\sqrt{2}}{2}$

$\cos x - \sin x = \sqrt{2}\sin(x-\frac{\pi}{4}) = \pm\frac{\sqrt{2}}{2}$ چرا این روش جواب غلط میده

$a\sin x \pm b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\sin(x\pm\alpha) \to \tan = \frac{b}{a}$

روش ۲

$1+\sin 2x = 2\cos^2 2x \quad 1+\sin 2x = 2-2\sin^2 2x \quad 2\sin^2 2x + \sin 2x - 1 = 0$

$(\sin 2x + \frac{2}{2})(\sin 2x - \frac{1}{2}) = 0$ → $\sin 2x = -1$, $\sin 2x = \frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{2} \quad x = k\pi - \frac{\pi}{4} \to x = \frac{3\pi}{4}$

$2x = 2k\pi + \frac{\pi}{6} \lor 2k\pi+\pi-\frac{\pi}{6}$ → $x = \frac{5\pi}{12} \to$ ~~[illegible]~~ غیرقابل قبول $\to \cos 2x = \cos\frac{5\pi}{6}$; $x = \frac{\pi}{12}$

جواب اولیه منتفی می‌شود و غیرقابل قبول

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$\frac{1-\tan x}{1+\tan x} = \tan 3x \quad \tan(\frac{\pi}{4}-x) = \tan 3x \quad 3x = \frac{\pi}{4} - x + k\pi$

$x = \frac{k\pi}{4} + \frac{\pi}{16}$

$\tan x \cancel{\#} + 1 \neq 0$

$\tan x \neq -1 \quad x \neq k\pi + \frac{\pi}{4}$

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الف) $\sin^4 x - \cos^4 x = 1 \quad \sin^2 x - \cos^2 x = 1 \quad -\cos 2x = 1 \quad \cos 2x = -1$

$2x = 2k\pi + \pi \quad x = k\pi + \frac{\pi}{2} \xrightarrow{[0,2\pi]} \frac{\pi}{2}, \frac{3\pi}{2}$ ②جواب

ب) $\sin^3 x - \cos^3 x = -1 \quad (\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x\cos x) = -1$

$(\sin x - \cos x)(1 + \sin x\cos x) = -1$ ۹.۸