۱ـ بینا فرقی

$\cos 2x = 3\sin x - 1 \rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \dfrac{-3 \pm \sqrt{25}}{4} = \dfrac{-3 \pm 5}{4} = -2$ غ‌ق $/ \dfrac{1}{2}$

$\rightarrow \sin x = \sin\dfrac{\pi}{6} \rightarrow x = 2k\pi + \dfrac{\pi}{6} \qquad x = 2k\pi + \dfrac{5\pi}{6}$

۱۹٫۲۵ آفرین

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ب) $2\cos^2 x + \cos x - 3 = 0 \rightarrow \cos x = \dfrac{-1 \pm 5}{4} = 1$ و $-\dfrac{3}{2}$ غ‌ق

$\cos x = 1 \rightarrow x = 2k\pi$

۲ـ

الف) $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x = -\cos 2x = 2\sin 2x\cos 2x$

$2\sin 2x\cos 2x + \cos 2x = 0 \rightarrow \cos 2x(2\sin 2x + 1) = 0 \rightarrow \cos 2x = 0 \rightarrow \sin 2x = -\dfrac{1}{2}$

$\cos 2x = 0 \rightarrow 2x = k\pi + \dfrac{\pi}{2} \rightarrow x = \dfrac{k\pi}{2} + \dfrac{\pi}{4}$

$\sin 2x = -\dfrac{1}{2} = \sin(-\dfrac{\pi}{6}) \rightarrow 2x = 2k\pi - \dfrac{\pi}{6} \rightarrow x = k\pi - \dfrac{\pi}{12}$

$\rightarrow 2x = 2k\pi + \dfrac{7\pi}{6} \rightarrow x = k\pi + \dfrac{7\pi}{12}$

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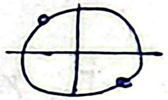
ب) $2\cos^2 x - 1 + 2\sin x\cos x = 0 \rightarrow \cos 2x + \sin 2x = 0$



$2x = k\pi - \dfrac{\pi}{4} \rightarrow x = \dfrac{k\pi}{2} - \dfrac{\pi}{8}$

۳ـ

۱) $\dfrac{\cos x\,(2\cos x + 1)}{\sin x} = \dfrac{1}{\sin x} \rightarrow \sin x \neq 0 \rightarrow 2\cos^2 x + \cos x - 1 = 0$

$\cos x = \dfrac{-1 \pm 3}{4} = -1$ و $\dfrac{1}{2} \rightarrow \cos x = -1 \rightarrow$ غ‌ق

چون $\sin x = 0$ می‌شود

$\cos x = \dfrac{1}{2} = \cos(\dfrac{\pi}{3}) \rightarrow x = 2k\pi \pm \dfrac{\pi}{3}$

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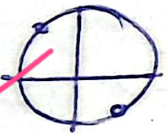
ب)

$$\rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$$

$$\cancel{2\sin^2 x} - \sin x\cos x + \cos^2 x - \cancel{2\sin^2 x} - 2\cos^2 x = 0$$

$$+\cos^2 x + \sin x\cos x = 0 \qquad \cos x(\cos x + \sin x) = 0$$

$$\cos x = 0 \rightarrow x = k\pi + \frac{\pi}{2} \qquad \cos x = -\sin x$$



$$\rightarrow x = k\pi - \frac{\pi}{4}$$

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \rightarrow +\sin(x-\frac{\pi}{4})\cos(x-\frac{\pi}{4}) = -\frac{1}{4}$

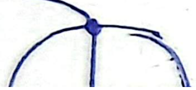
$\overbrace{\cos(x-\frac{\pi}{4}+\frac{\pi}{2})} = -\sin(x-\frac{\pi}{4})$

$\rightarrow \cancel{\tfrac{1}{2}(} \quad \frac{1}{2}\underbrace{\sin(2x-\frac{\pi}{2})}_{-\cos 2x} = -\frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3}$

$\rightarrow x = k\pi \pm \frac{\pi}{6}$

$L_0 = 18$

ب)



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ب) $\cos(۳x-\frac{\pi}{۹}) = -\cos(\frac{\pi}{۲}-۳x) = \cos(\pi-(\frac{\pi}{۲}-۳x))$

$\rightarrow \cos(۳x-\frac{\pi}{۹}) = \cos(\frac{\pi}{۲}+۳x) \rightarrow ۳x-\frac{\pi}{۹} = ۲k\pi+\frac{\pi}{۲}+۳x$ ✗

$\searrow ۳x-\frac{\pi}{۹} = ۲k\pi-۳x-\frac{\pi}{۲} \rightarrow ۶x = ۲k\pi-\frac{۷\pi}{۱۸}$

$\rightarrow x = \frac{k\pi}{۳} - \frac{۷\pi}{۷۲}$

ب)

ـ۵

$$\frac{۲\sin ۲x\cos ۲x + \sin ۲x}{\sin ۲x} = \cos ۲x \to ۲\sin ۲x \cos ۲x + \sin ۲x = \sin ۲x \cos ۲x$$

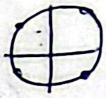
$$\to \sin ۲x \cos ۲x + \sin ۲x = ۰ \to \sin ۲x(\cos ۲x + 1) = ۰ \to \sin ۲x = ۰$$ غ ق ق

$$\cos ۲x = -1 \to ۲x = ۲k\pi + \pi \to x = k\pi + \frac{\pi}{۲}$$ (۱,۵)

ـ۶

$$\frac{\sin\frac{x}{۲}}{1+\cos\frac{x}{۲}} = \tan\frac{x}{۴} \qquad \frac{1}{\sin\frac{x}{۲}} + \frac{\cos\frac{x}{۲}}{\sin\frac{x}{۲}} = \frac{1+\cos\frac{x}{۲}}{\sin\frac{x}{۲}} = \cot\frac{x}{۴}$$ (۲)

$$\to \tan\frac{x}{۴} = \frac{1}{\tan\frac{x}{۴}} \to \tan\frac{x}{۴} = \pm 1$$



$$\frac{x}{۴} = \frac{k\pi}{۲} + \frac{\pi}{۴}$$

$$\to x = ۲k\pi + \pi \to x = \pi \text{ و } -\pi \to ۲\pi$$

ـ۷

$$\cos^۲ x - \sin^۲ x \cos ۳x = 1 \to -\sin^۲ x \cos ۳x = \sin^۲ x$$

$$\to \sin^۲ x(1 + \cos ۳x) = ۰ \to \sin x = ۰ \to x = k\pi$$ (۲)

$$\cos ۳x = -1 \to ۳x = ۲k\pi + \pi \to x = \frac{۲k\pi}{۳} + \frac{\pi}{۳} \to x = ۰ \text{ و } \pi \text{ و } ۲\pi ,\ \frac{\pi}{۳} \text{ و } \frac{۵\pi}{۳} \to ۵$$

ـ۸

$$\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{۴} - x\right) = \tan ۳x \to$$

$$۳x = k\pi + \frac{\pi}{۴} - x \to ۴x = k\pi + \frac{\pi}{۴} \to x = \frac{k\pi}{۴} + \frac{\pi}{۱۶}$$ (۲)

ـ۹

$1+\sin ۲x$ همواره + $\to$ $\underbrace{1}_{\sin^۲ ۲x + \cos^۲ ۲x} + \sin ۲x = ۲\cos^۲ ۲x \to \underbrace{\sin^۲ ۲x - \cos^۲ ۲x}_{-\cos ۴x} + \sin ۲x = ۰$ (۱,۷۵)

$$\to \sin ۲x = \cos ۴x \to \cos ۴x = \cos\left(\frac{\pi}{۲} - ۲x\right)$$

$$\to ۴x = ۲k\pi + \frac{\pi}{۲} - ۲x \to ۶x = ۲k\pi + \frac{\pi}{۲} \to x = \frac{k\pi}{۳} + \frac{\pi}{۱۲}$$

$$۴x = ۲k\pi - \frac{\pi}{۲} + ۲x \to ۲x = ۲k\pi - \frac{\pi}{۲} \to x = k\pi - \frac{\pi}{۴}$$

$x = \frac{\pi}{۱۲} \text{ و } \cancel{\frac{۵\pi}{۱۲}} \text{ و } \cancel{\frac{۹\pi}{۱۲}} \text{ و } \frac{۳\pi}{۴}$ به ازای $x = \frac{۵\pi}{۱۲}$ $\cos ۲x$ منفی می‌شود / چون جواب رادیکال است غ ق ق است

$$\frac{۹\pi}{۱۲} = \frac{۳\pi}{۴}$$

۱-

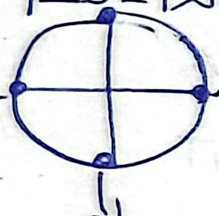
الف) $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x) = -\cos 2x = 1$

$\cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{2}$ و $\frac{3\pi}{2}$

۲

ب)

$1 - 0 = 1 \times$



$0 + 1 = 1 \times$ $\quad$ $0 - 1 = -1 \checkmark$

$-1 - 0 = -1 \checkmark$

0 و $\frac{3\pi}{2}$ و 2π

سوال ۵

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \overbrace{\sin^2 x + \cos^2 x}^{1}$$

$$\frac{\cancel{\sin 2x}(2\cos 2x + 1)}{\cancel{\sin 2x}} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \longrightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \longrightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$