۱۹٫۵ آفرین خیلی خوب نوشتی :)

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دوازدهم هفته _ B

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۱) الف) $\underbrace{\cos 2x}_{1-2\sin^2 x} = 3\sin x - 1 \Rightarrow 2\sin^2 + 3\sin x - 2 = 0$

$x = \dfrac{-3 \pm 5}{4} \left\langle \begin{array}{l} -2 \ \times \\ \frac{1}{2} = \sin\frac{\pi}{6} \end{array} \right. \Rightarrow \boxed{x = \left\langle \begin{array}{l} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \frac{5\pi}{6} \end{array} \right.}$

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ب) $\underbrace{\cos 2x}_{2\cos^2 - 1} + \cos x - 2 = 0 \rightarrow 2\cos^2 + \cos - 3 = 0 \Rightarrow x = \left\langle \begin{array}{l} 1 \ \checkmark \\ -\frac{3}{2} \ \times \end{array} \right.$

$\cos x = \cos \pi \Rightarrow x = 2k\pi \pm 2\pi = \boxed{2k\pi}$

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۲) الف) $\underbrace{\sin^4 x - \cos^4 x}_{\cancel{(\sin^2 + \cos^2)}(\sin^2 - \cos^2)} = \sin 4x \rightarrow -\cos 2x = \sin 4x \rightarrow$

$\cos 2x + \sin 4x = 0 \rightarrow \cos 2x(1 + 2\sin 2x) = 0$

$$\Rightarrow \begin{cases} \cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \rightarrow x = k\pi \pm \frac{\pi}{4} \\ \sin 2x = -\frac{1}{2} \rightarrow 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \\ \qquad\qquad\quad\ \ 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \end{cases}$$

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ب) $2\cos^2 x + 2\sin x \cos x = 1 \rightarrow \cos 2x = -\sin 2x$

$\rightarrow \cos 2x = \sin(-2x) \rightarrow \cos 2x = \cos\left(\frac{\pi}{2} + 2x\right) \rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x \ \times$ جواب ندارد

$2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow$

$4x = 2k\pi - \frac{\pi}{2} \rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

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۳) الف) $\cot(2\cos x + 1) = \dfrac{1}{\sin x} \xrightarrow{\sin x \neq 0} \cos x(2\cos x + 1) = 1$

$\rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow x \left\langle \begin{array}{l} -1 \rightarrow 2k\pi \pm \pi \not\Rightarrow \sin x = 0 \Rightarrow \text{غ‌ق‌ق} \\ \frac{1}{2} \rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}} \end{array} \right.$

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ب) $1 - \cos 2x - \frac{1}{2}\sin 2x + \frac{1}{2} + \frac{1}{2}\cos 2x = 2 \rightarrow$

$\cos 2x + \sin 2x + 1 = 0 \rightarrow \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right) = -1 \rightarrow \sin\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\rightarrow 2x + \frac{\pi}{4} = \left\langle \begin{array}{l} 2k\pi + \frac{5\pi}{4} \rightarrow 2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2} \\ 2k\pi - \frac{\pi}{4} \rightarrow 2k\pi - \frac{\pi}{2} \rightarrow x = k\pi - \frac{\pi}{4} \end{array} \right.$

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \to -\sin(x-\frac{\pi}{4})\cos(x-\frac{\pi}{4})=\frac{1}{4}$ $(\times 4$

$\sin(2x-\frac{\pi}{2})=-\frac{1}{2} \to -\cos 2x=-\frac{1}{2} \to \cos 2x=\frac{1}{2} \to \boxed{k\pi\pm\frac{\pi}{6}}$

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ب) $\cos(2x-\frac{\pi}{9})=\sin(-2x) \to \sin(\frac{\pi}{2}-2x+\frac{\pi}{9})=\sin(-2x)$

$\to \sin(\frac{11\pi}{18}-2x)=\sin(-2x) \to \frac{11\pi}{18}-2x=2k\pi+\pi+2x \to$

$-4x=2k\pi+\frac{7\pi}{18} \to -x=\frac{k\pi}{2}+\frac{7\pi}{72} \Rightarrow x=-\frac{k\pi}{2}-\frac{7\pi}{72}$

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۵) $\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \to 1=2\cos 2x+1$

$\xrightarrow[\sin 2x\neq 0]{x\neq\frac{k\pi}{2}} \cos 2x=0 \to \boxed{x=k\pi\pm\frac{\pi}{4}}$

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۶) $\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \xrightarrow[\sin\frac{x}{2}\neq 0]{\cos\frac{x}{2}\neq -1} \sin^2=1+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \to$

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$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \to 2\cos\frac{x}{2}(\cos\frac{x}{2}+1)=0 <$ $\cos\frac{x}{2}=0 \to$; $\cos\frac{x}{2}=-1$ ×

$\cos\frac{x}{2}=0 \to \frac{x}{2}=2k\pi\pm\frac{\pi}{2} \to \underbrace{x=4k\pi\pm\pi} \xrightarrow{[-\pi,\frac{5\pi}{4}]} \{-\pi,\pi\}$

$\Rightarrow |\pi-(-\pi)|=2\pi$

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۷) $\cancel{1}-\sin^2-\sin^2 x\cos 3x=\cancel{1} \to \sin^2 x(1+\cos 3x)=0$

$\sin^2 x=0 \to \underbrace{k\pi}$, $\cos 3x=-1 \to 3x=2k\pi\pm\pi \to x=\underbrace{\frac{2k\pi}{3}\pm\frac{\pi}{3}}$

$\Rightarrow$ جواب‌ها $=\{0,\pi,2\pi,\frac{\pi}{3},\frac{5\pi}{3}\} \Rightarrow$ $\boxed{\text{۵ جواب}}$

$D=[0,2\pi]$

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۸) $\frac{1-\tan x}{1+\tan x}=\tan 3x \to \tan(\frac{\pi}{4}-x)=\tan 3x \to 3x=k\pi+\frac{\pi}{4}-x$

$\to 4x=k\pi+\frac{\pi}{4} \to \boxed{\frac{k\pi}{4}+\frac{\pi}{16}=x}$

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(۹ $\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \longrightarrow 1+\sin 2x = 2\cos^2 2x$

$\Rightarrow 1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$

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$2x = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$

$\longrightarrow 2x = 2k\pi - \frac{\pi}{2} \to x = k\pi - \frac{\pi}{4}$

$\longrightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{6} \to k\pi + \frac{\pi}{12} \\ 2x = 2k\pi + \frac{5\pi}{6} \to k\pi + \frac{5\pi}{12} \end{cases} \xrightarrow{[0,\pi]} \left\{\frac{3\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}\right\}$

۳ جواب

به ازای $x = \frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، چون جواب را اشکال است نوشته است

(۱۰ الف) $\sin^4 x - \cos^4 x = 1 \to -\cos 2x = 1 \to \cos 2x = -1$

$\to 2x = 2k\pi \pm \pi \to \boxed{x = k\pi \pm \frac{\pi}{2}} \xrightarrow{[0,2\pi]} \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

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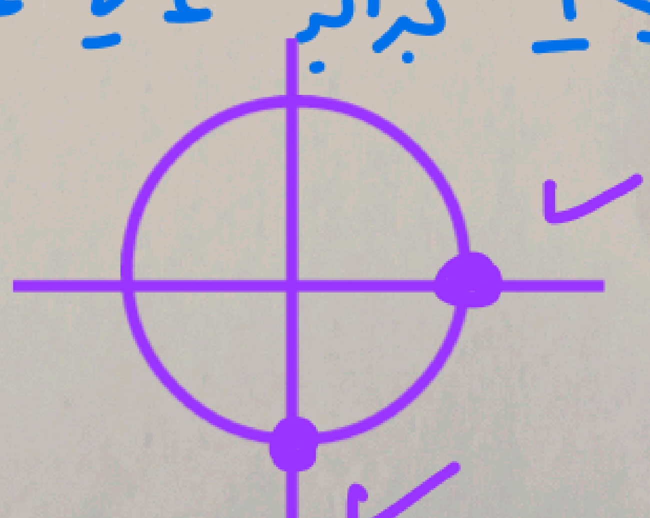
ب) $\sin^3 x - \cos^3 x = -1 \to \begin{cases} \sin^3 x = 0 , \cos^3 = 1 \\ \sin^3 x = -1 , \cos^3 = 0 \end{cases}$

$\to \left\{2\pi , \frac{3\pi}{2}\right\} , 0$

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1. فقط کافی است نقاط اصلی را چک کنی!

$\sin^3 x - \cos^3 x = 1$



$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

سوال ۹

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$

$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1$ یا $\sin 2x = +\frac{1}{2}$

$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4}$ ✓

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12}$ ✓ یا $\frac{5\pi}{12}$ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است مثبت است

* معادله ۲ جواب دارد