

سوال ۱) $\cos 2x = 3 \sin x - 1 \rightarrow 1 - 2 \sin^2 x = 3 \sin x - 1 \rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0 \rightarrow (\sin x + 2)(2 \sin x - 1) = 0$
 $\downarrow$
 $\sin x = -2$ غلط
 $\rightarrow \sin x = \frac{1}{2} \Rightarrow x = \left\{ 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right\}$

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2 \cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2 \cos^2 x + \cos x - 3 = 0$
 $\rightarrow \cos x = 1 \rightarrow x = 2k\pi$
 $\rightarrow \cos x = -\frac{3}{2}$ غلط

سوال ۲) $\sin^4 x - \cos^4 x = \sin^2 x \rightarrow -\cos 2x = 2 \sin^2 x \cos 2x \rightarrow \cos 2x = 0 \rightarrow 2x = 2k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{4}$

$\cos 2x \neq 0 \rightarrow \sin 2x = -\frac{1}{2} \rightarrow 2x \in \left\{ 2k\pi + \frac{7\pi}{6}, 2k\pi + \frac{11\pi}{6} \right\} \Rightarrow x \in \left\{ k\pi + \frac{7\pi}{12}, k\pi + \frac{11\pi}{12} \right\}$

ب) $2 \cos^2 x + 2 \sin x \cos x = 1 \rightarrow \sin 2x = 1 - 2 \cos^2 x \rightarrow \sin 2x = -\cos 2x \rightarrow \tan 2x = -1 \rightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

سوال ۳) $\cot x (2 \cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{\cos x}{\sin x} (2 \cos x + 1) = \frac{1}{\sin x} \Rightarrow 2 \cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = -1 \rightarrow x = 2k\pi + \pi$
 $\downarrow$
 $\cos x = \frac{1}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$

ب) $2 \sin^2 x - \sin x \cos x + \cos^2 x = 2 \xrightarrow{\div \sin^2 x} 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x) \rightarrow \cot^2 x + \cot x = 0 \rightarrow \cot x = 0 \rightarrow x = k\pi + \frac{\pi}{2}$
 $\downarrow$
 $\cot x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

سوال ۴) $\cos\left(\frac{\pi}{4} + x\right) \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{4} \xrightarrow{\text{در زاویه ی متمم}} \cos\left(\frac{\pi}{4} + x\right) \cos\left(\frac{\pi}{4} - x\right) = \frac{1}{4} \rightarrow \sin\left(\frac{\pi}{4} + x\right) \cos\left(\frac{\pi}{4} + x\right) = \frac{1}{4} \rightarrow \frac{1}{2} \sin\left(\frac{\pi}{2} + 2x\right) = \frac{1}{4}$

$\rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{3} \rightarrow x = k\pi + \frac{\pi}{6}$

ب) $\cos\left(2x - \frac{\pi}{4}\right) = -\sin 2x \rightarrow \cos\left(2x - \frac{\pi}{4}\right) = \sin(-2x) \xrightarrow{\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta} \cos\left(2x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} - (-2x)\right) \rightarrow \cos\left(2x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} + 2x\right)$

$2x - \frac{\pi}{4} = 2k\pi + 2x + \frac{\pi}{2} \rightarrow -\frac{\pi}{4} = 2k\pi + \frac{\pi}{2}$

$2x - \frac{\pi}{4} = 2k\pi - 2x - \frac{\pi}{2} \rightarrow 4x = 2k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

سوال ۵) $\frac{\sin^2 x + \sin^2 x}{\sin^2 x} - \sin^2 x = \cos^2 x \rightarrow \frac{2 \sin^2 x \cos^2 x + \sin^2 x}{\sin^2 x} = \cos^2 x + \sin^2 x \rightarrow \frac{\sin^2 x (2 \cos^2 x + 1)}{\sin^2 x} = 1 \rightarrow \cos^2 x = 0$

$2x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}, \sin^2 x \neq 0 \rightarrow 2x \neq k\pi \rightarrow x \neq \frac{k\pi}{2}$

سوال ۶) $\frac{\sin \frac{\pi}{2}}{1 + \cos \frac{\pi}{2}} = \frac{1}{\sin \frac{\pi}{2}} + \cot \frac{\pi}{2} \rightarrow \frac{\sin \frac{\pi}{2}}{1 + \cos \frac{\pi}{2}} = \frac{1}{\sin \frac{\pi}{2}} + \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} \rightarrow \sin^2 \frac{\pi}{2} = (1 + \cos \frac{\pi}{2})^2 \rightarrow \sin^2 \frac{\pi}{2} = \cos^2 \frac{\pi}{2} + 2 \cos \frac{\pi}{2} + 1 \rightarrow \sin^2 \frac{\pi}{2} + \cos^2 \frac{\pi}{2}$
 $\rightarrow 2 \cos^2 \frac{\pi}{2} + 2 \cos \frac{\pi}{2} = 0 \rightarrow \cos \frac{\pi}{2} = -1 \rightarrow$ در این بازه $\cos \frac{\pi}{2}$ برابر ۱ نمی شود

$\cos \frac{\pi}{2} = 0 \rightarrow \frac{\pi}{2} = \pm \frac{\pi}{2} \rightarrow x = \pm \pi \rightarrow |\pi - (-\pi)| = 2\pi$

$$\cos^2 n - \sin^2 n \cos 2n = 1 \rightarrow 1 - \cos^2 n = -\sin^2 n \cos 2n \rightarrow \sin^2 n = -\sin^2 n \cos 2n$$

$$[0, 2\pi]$$

سوال ۷

$$\rightarrow \sin^2 n = 0 \rightarrow n = \pi, 2\pi$$

$$\rightarrow \sin^2 n \neq 0 \rightarrow \cos 2n = -1 \rightarrow 2n = \pi \rightarrow n = \frac{\pi}{2}$$

$$\frac{1 - \tan n}{1 + \tan n} = \tan 2n \rightarrow \frac{\tan \frac{\pi}{4} - \tan n}{1 + \tan \frac{\pi}{4} \tan n} = \tan 2n \rightarrow \tan(\frac{\pi}{4} - n) = \tan 2n \rightarrow 2n = k\pi + \frac{\pi}{4} - n \rightarrow 3n = k\pi + \frac{\pi}{4}$$

سوال ۸

$$n = \frac{k\pi}{3} + \frac{\pi}{12}$$

۱- $\tan n \neq -1$ در مجموعی جواب تغییری به وجود نمی آورد.

$$\sqrt{1 + \sin 2n} = \sqrt{2} \cos n \rightarrow \sqrt{\sin^2 n + \cos^2 n + 2\sin n \cos n} = \sqrt{2} \cos n \rightarrow \sqrt{(\sin n + \cos n)^2} = \sqrt{2} \cos n \xrightarrow{\sin n + \cos n > 0}$$

سوال ۹

$$\sin n + \cos n = \sqrt{2} (\cos n + \sin n) (\cos n - \sin n) \rightarrow \cos n - \sin n = \frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(n + \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow \sin(n + \frac{\pi}{4}) = -\frac{1}{2}$$

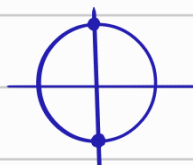
$$n + \frac{\pi}{4} = \frac{\sqrt{2}}{4} \rightarrow n = \frac{\sqrt{2}}{4} - \frac{\pi}{4} = \frac{2\pi}{11} \checkmark \quad \text{یا} \quad n + \frac{\pi}{4} = \frac{11\pi}{4} \Rightarrow n = \frac{11\pi}{4} - \frac{\pi}{4} = \frac{10\pi}{4} \checkmark$$

$$\xrightarrow{\sin n + \cos n < 0} \sin n + \cos n = -\sqrt{2} (\cos n + \sin n) (\cos n - \sin n) \rightarrow \cos n - \sin n = -\frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(n + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \rightarrow \sin(n + \frac{\pi}{4}) = \frac{1}{2}$$

$$n + \frac{\pi}{4} = \frac{\pi}{4} \rightarrow n = \frac{\pi}{4} - \frac{\pi}{4} = -\frac{\sqrt{2}}{11} \checkmark \quad \text{یا} \quad n + \frac{\pi}{4} = \frac{2\pi}{4} \rightarrow n = \frac{2\pi}{4} - \frac{\pi}{4} = \frac{\pi}{4} \checkmark$$

$$\xrightarrow{\sin n + \cos n = 0} \sin n = -\cos n \rightarrow n = \frac{3\pi}{4} \checkmark \quad n = \frac{5\pi}{4} \checkmark$$

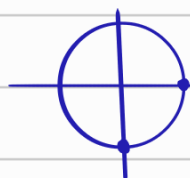
$$\text{الف) } \sin^2 n - \cos^2 n = 1$$



$$\rightarrow n = \frac{\pi}{2}, \frac{3\pi}{2}$$

سوال ۱۰ چهار نقطه اصلی در استوانه ی سین:

$$\text{ب) } \sin^2 n - \cos^2 n = -1$$



$$\rightarrow n = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$