

سوال ۱) $\cos 2x = 3 \sin x - 1 \rightarrow 1 - 2 \sin^2 x = 3 \sin x - 1 \rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0 \rightarrow (\sin x + 2)(2 \sin x - 1) = 0$
 $\downarrow$
 $\sin x = -2$ غلط $\sin x = \frac{1}{2}$
 $\Rightarrow x = \{2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\}$ ✓

ب) $\cos 2x + \cos x - 2 = 0 \rightarrow 2 \cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2 \cos^2 x + \cos x - 3 = 0$
 $\rightarrow \cos x = 1 \rightarrow x = 2k\pi$ ✓
 $\rightarrow \cos x = -\frac{3}{2}$ غلط

سوال ۲) $\sin^2 x - \cos^2 x = \sin 2x \rightarrow -\cos 2x = 2 \sin x \cos x \rightarrow \cos 2x = 0 \rightarrow 2x = 2k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{4}$

$\cos 2x \neq 0 \rightarrow \sin 2x = -\frac{1}{2} \rightarrow 2x \in \{2k\pi + \frac{7\pi}{6}, 2k\pi + \frac{11\pi}{6}\} \Rightarrow x \in \{k\pi + \frac{7\pi}{12}, k\pi + \frac{11\pi}{12}\}$ ✓

ب) $2 \cos^2 x + 2 \sin x \cos x = 1 \rightarrow \sin 2x = 1 - 2 \cos^2 x \rightarrow \sin 2x = -\cos 2x \rightarrow \tan 2x = -1 \rightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$ ✓

سوال ۳) $\cot x (2 \cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{\cos x}{\sin x} (2 \cos x + 1) = \frac{1}{\sin x} \Rightarrow 2 \cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = -1 \rightarrow x = 2k\pi + \pi$ ✗
 $\downarrow$
 $\cos x = \frac{1}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$ ✓

ب) $2 \sin^2 x - \sin x \cos x + \cos^2 x = 2 \div \sin^2 x \rightarrow 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x) \rightarrow \cot^2 x + \cot x = 0 \rightarrow \cot x = 0 \rightarrow x = k\pi + \frac{\pi}{2}$
 $\downarrow$
 $\cot x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$ ✓

آنها $\cos x = 0$ باشد $\sin x = 0$ است و عبارات تعریف نشده هستند
 پس جواب غیر قابل قبول محسوب میشوند

سوال ۴) $\cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{4}) = \frac{1}{4} \rightarrow \cos(\frac{\pi}{4} + x) \cos(\frac{\pi}{4} - x) = \frac{1}{4} \rightarrow \sin(\frac{\pi}{4} + x) \cos(\frac{\pi}{4} + x) = \frac{1}{4} \rightarrow \frac{1}{2} \sin(\frac{\pi}{2} + 2x) = \frac{1}{4}$
 $\rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$ ✓

ب) $\cos(x - \frac{\pi}{4}) = -\sin x \rightarrow \cos(x - \frac{\pi}{4}) = \sin(-x) \xrightarrow{\cos(\frac{\pi}{2} - \theta) = \sin \theta} \cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{2} - (-x)) \rightarrow \cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{2} + x)$
 $x - \frac{\pi}{4} = 2k\pi + x + \frac{\pi}{2} \rightarrow -\frac{\pi}{4} = 2k\pi + \frac{\pi}{2}$ ✓
 $x - \frac{\pi}{4} = 2k\pi - x - \frac{\pi}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{4} \rightarrow x = k\pi - \frac{\pi}{8}$

سوال ۵) $\frac{\sin 2x + \sin x}{\sin x} - \sin^2 x = \cos^2 x \rightarrow \frac{2 \sin x \cos x + \sin x}{\sin x} = \cos^2 x + \sin^2 x \rightarrow \frac{\sin x (2 \cos x + 1)}{\sin x} = 1 \rightarrow \cos x = 0$
 $x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$, $\sin x \neq 0 \rightarrow x \neq k\pi$ ✓

سوال ۶) $\frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}} = \frac{1}{\sin \frac{\pi}{4}} + \cot \frac{\pi}{4} \rightarrow \frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}} = \frac{1}{\sin \frac{\pi}{4}} + \frac{\cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} \rightarrow \sin^2 \frac{\pi}{4} = (1 + \cos \frac{\pi}{4})^2 \rightarrow \sin^2 \frac{\pi}{4} = \cos^2 \frac{\pi}{4} + 2 \cos \frac{\pi}{4} + 1$
 $\rightarrow 2 \cos^2 \frac{\pi}{4} + 2 \cos \frac{\pi}{4} = 0 \rightarrow \cos \frac{\pi}{4} = -1 \rightarrow$ در این بازه $\cos \frac{\pi}{4}$ برابر -1 نمی شود
 $\cos \frac{\pi}{4} = 0 \rightarrow \frac{\pi}{4} = \pm \frac{\pi}{2} \rightarrow x = \pm \pi \rightarrow |\pi - (-\pi)| = 2\pi$ ✓

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \rightarrow 1 - \cos^2 x = -\sin^2 x \cos^2 x \rightarrow \sin^2 x = -\sin^2 x \cos^2 x$$

$[0, \pi]$

سوال ۷

$$\rightarrow \sin^2 x = 0 \rightarrow x = \pi, 0$$

۱, ۲۵

$$\rightarrow \sin^2 x \neq 0 \rightarrow \cos^2 x = -1 \rightarrow x = \pi \rightarrow x = \frac{\pi}{2}$$

$$\frac{1 - \tan x}{1 + \tan x} = \tan^2 x \rightarrow \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} = \tan^2 x \rightarrow \tan(\frac{\pi}{4} - x) = \tan^2 x \rightarrow x = k\pi + \frac{\pi}{4} - x \rightarrow x = k\pi + \frac{\pi}{4}$$

سوال ۸

$$x = \frac{k\pi}{4} + \frac{\pi}{14}$$

۱- $\tan x \neq -1$ در مجموعه جواب تغییری به وجود نمی آورد.

۲

$$\sqrt{1 + \sin^2 x} = \sqrt{2} \cos x \rightarrow \sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} = \sqrt{2} \cos x \rightarrow \sqrt{(\sin x + \cos x)^2} = \sqrt{2} \cos x \rightarrow \sin x + \cos x = 0$$

سوال ۹

$$\sin x + \cos x = \sqrt{2} (\cos x + \sin x) (\cos x - \sin x) \rightarrow \cos x - \sin x = \frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \rightarrow \sin(x + \frac{\pi}{4}) = -\frac{1}{2}$$

$$x + \frac{\pi}{4} = \frac{\sqrt{2}}{4} \rightarrow x = \frac{\sqrt{2}}{4} - \frac{\pi}{4} = \frac{2\pi}{18} \checkmark \quad x + \frac{\pi}{4} = \frac{11\pi}{4} \Rightarrow x = \frac{11\pi}{4} - \frac{\pi}{4} = \frac{10\pi}{4} \checkmark$$

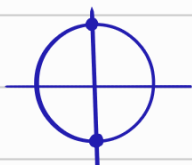
$$\sin x + \cos x = 0 \rightarrow \sin x + \cos x = -\sqrt{2} (\cos x + \sin x) (\cos x - \sin x) \rightarrow \cos x - \sin x = -\frac{\sqrt{2}}{2} \rightarrow -\sqrt{2} \sin(x + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \rightarrow \sin(x + \frac{\pi}{4}) = \frac{1}{2}$$

$$x + \frac{\pi}{4} = \frac{\pi}{4} \rightarrow x = \frac{\pi}{4} - \frac{\pi}{4} = -\frac{\sqrt{2}}{18} \checkmark \quad x + \frac{\pi}{4} = \frac{5\pi}{4} \rightarrow x = \frac{5\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2} \checkmark$$

۲

$$\sin x + \cos x = 0 \rightarrow \sin x = -\cos x \rightarrow x = \frac{3\pi}{4} \checkmark \quad x = \frac{5\pi}{4} \checkmark$$

$$\sin^2 x - \cos^2 x = 1$$



$$\rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

سوال ۱۰ چهار نقطه اصلی در استخوانی است:

$$\sin^2 x - \cos^2 x = -1$$



$$\rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

۲

سوال ۷

$$\cos^2 x - \sin^2 x \cos^2 x = 1 \quad \cos^2 x = 1 - \sin^2 x$$

$$1 - \sin^2 x - \sin^2 x \cos^2 x = 1 \rightarrow \sin^2 x (1 + \cos^2 x) = 0 \rightarrow \sin x = 0$$

$$\sin x = 0 \rightarrow x = k\pi \rightarrow x = 0, \pi, 2\pi$$

$$\rightarrow \cos^2 x = 1$$

$$\cos^2 x = -1 \rightarrow x = k\pi + \pi \rightarrow x = \frac{k\pi + \pi}{2} \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

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