سوال ۱

الف) $\cos 2\alpha - 2\sin\alpha = 1 \to 1 - 2\sin^2\alpha - 2\sin\alpha = 1$

$2\sin^2\alpha + 2\sin\alpha = 0 \to \sin^2\alpha + \sin\alpha = 0 \to \sin\alpha(\sin\alpha + 1) = 0$

$\sin\alpha = 0 \to \alpha = k\pi$ ، $\sin\alpha = -1 \to \alpha = 2k\pi + \frac{3\pi}{2}$

ب) $\cos 2\alpha + \cos\alpha = 0 \Rightarrow 2\cos^2\alpha - 1 + \cos\alpha = 0 \to$

$2\cos^2\alpha + \cos\alpha - 1 = 0 \to \cos^2\alpha + \cos\alpha - 1 = 0 \to (\cos\alpha + 1)(\cos\alpha - \frac{1}{2}) = 0$

$\cos\alpha = -1 \to \alpha = 2k\pi + \pi$

سوال ۲

الف) $\sin^4\alpha - \cos^4\alpha = \sin 4\alpha \to (\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha) = \sin 4\alpha$

$-\cos 2\alpha = \sin 4\alpha \Rightarrow \sin(-4\alpha) = \cos 2\alpha \to \cos(\frac{\pi}{2} + 4\alpha) = \cos 2\alpha$

$\frac{\pi}{2} + 4\alpha = 2k\pi + 2\alpha \to 2\alpha = 2k\pi - \frac{\pi}{2} \to \alpha = k\pi - \frac{\pi}{4}$

$\frac{\pi}{2} - 4\alpha = 2k\pi - 2\alpha \to -2\alpha = 2k\pi - \frac{\pi}{2} \to \alpha = \frac{\pi}{4} - k\pi$

ب) $2\cos^2\alpha + 2\sin\alpha\cos\alpha = 1 \to 2\sin\alpha\cos\alpha = 1 - 2\cos^2\alpha \to \sin 2\alpha = -\cos 2\alpha$

$\to \cos(\frac{\pi}{2} - 2\alpha) = \cos(\pi - 2\alpha) \to \frac{\pi}{2} - 2\alpha = 2k\pi + \pi - 2\alpha$ غیرقابل قبول

$2\alpha - \frac{\pi}{2} = 2k\pi - 2\alpha \to 4\alpha = 2k\pi + \frac{\pi}{2} \to \alpha = \frac{k\pi}{2} + \frac{\pi}{8}$

سوال ۳

الف) $\frac{\cot\alpha}{\sin^2\alpha}(2\cot\alpha + 1) = \frac{1}{\sin\alpha} \to \frac{2\cos^2\alpha + \cos\alpha}{\sin^3\alpha} = \frac{1}{\sin\alpha} \xrightarrow{\sin\alpha \neq 0}$

$2\cos^2\alpha + \cos\alpha - 1 = 0 \to \cos^2\alpha + \cos\alpha - 2 = 0 \to (\cos\alpha - 1)(\cos\alpha + 2)$

$\cos\alpha = 1$ $\quad \cos\alpha = -2$ غیرقابل قبول

$\cos\alpha = -1 \rightarrow \alpha = 2k\pi + \pi \rightarrow \sin = 0 \rightarrow$ غ ق ق

$\cot\alpha = \frac{1}{2} \rightarrow \alpha = 2k\pi + \frac{\pi}{3}$, $\alpha = 2k\pi - \frac{\pi}{3}$

د) $2\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha = 2$

$2(1-\cos^2\alpha) + \cos^2\alpha - \sin\alpha\cos\alpha = 2$

$2 - 2\cos^2\alpha + \cos^2\alpha - \sin\alpha\cos\alpha = 2$

$-\cos^2\alpha - \sin\alpha\cos\alpha = 0 \rightarrow \cos\alpha(\cos\alpha + \sin\alpha) = 0$

$\cos\alpha = 0 \rightarrow \alpha = 2k\pi + \frac{\pi}{2}$, $\alpha = 2k\pi - \frac{\pi}{2}$

$\sin(-\alpha) = \cos\alpha \rightarrow \cos\alpha = \cos(\frac{\pi}{2} + \alpha) \rightarrow \frac{\pi}{2} + \alpha = 2k\pi + \alpha$ غ ق ق

$\frac{\pi}{2} + \alpha = 2k\pi - \alpha \rightarrow 2\alpha = 2k\pi - \frac{\pi}{2} \rightarrow \alpha = k\pi - \frac{\pi}{4}$

سوال ۴

الف) $\cos(\frac{\pi}{4} + m)\cos(k\pi - \frac{\pi}{4}) = \frac{1}{4}$ $\rightarrow \frac{\pi}{4} + m + k\pi - m = \frac{\pi}{3}$

$\cos(\frac{\pi}{4} + m)\cos(\frac{\pi}{4} - m) = \cos(\frac{\pi}{2} + m)\sin(\frac{\pi}{4} + m) = \frac{1}{2}\sin(\frac{\pi}{2} + 2m) =$

$\frac{1}{2}\cos 2m = \frac{1}{4} \rightarrow \cos 2m = \frac{1}{2} \rightarrow 2m = 2k\pi + \frac{\pi}{3}$

$\rightarrow m = k\pi + \frac{\pi}{6}$

$\rightarrow 2m = 2k\pi - \frac{\pi}{3} \rightarrow m = k\pi - \frac{\pi}{6}$

$\cos(2m - \frac{\pi}{2}) = -\sin 2m \Rightarrow \sin(\frac{11\pi}{12} - 2m) = \sin(-2m)$

$2x = 2k\pi + 2x - \frac{11\pi}{12}$ غق

$\sin(2x) = \sin\left(2x - \frac{11\pi}{12}\right) \xrightarrow{1} 2x = 2k\pi + \pi - 2x + \frac{11\pi}{12}$

$4x = 2k\pi + \pi + \frac{11\pi}{12} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4} + \frac{11\pi}{48}$

سوال ۳

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \sin 2x \neq 0$

$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \cos^2 x + \sin^2 x \rightarrow 2\cos 2x + 1 = 1$

$\cos 2\alpha = 0 \rightarrow 2\alpha = 2k\pi + \frac{\pi}{2} \rightarrow \alpha = k\pi + \frac{\pi}{4}$

$2\alpha = 2k\pi - \frac{\pi}{2} \rightarrow \alpha = k\pi - \frac{\pi}{4}$

سوال ۴

$\sin\frac{x}{2} \neq 0 \ , \ \cos\frac{x}{2} \neq -1$

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} - \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \xrightarrow{\frac{x}{2}=\alpha} \frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$

$\sin^2\alpha = (1+\cos\alpha)^2 \rightarrow 1 - \cos^2\alpha = 1 + \cos^2\alpha + 2\cos\alpha$

$2\cos^2\alpha + 2\cos\alpha = 0 \rightarrow 2\cos\alpha(\cos\alpha + 1) = 0 \rightarrow \cos\alpha = 0$

$\rightarrow \cos\alpha = -1$ غق

$\cos\frac{x}{2} = 0 \rightarrow \frac{x}{2} = 2k\pi + \frac{\pi}{2} \rightarrow x = 4k\pi + \pi \rightarrow \pi$

$\frac{x}{2} = 2k\pi - \frac{\pi}{2} \rightarrow x = 4k\pi - \pi \rightarrow -\pi$

$|\pi - (-\pi)| = 2\pi$

$\cos^2 x - \sin^2 x \cos 2x = 1$ سوال ۷

$2 - \sin^2 x - \sin^2 x (\cos 2x = 2 \rightarrow \sin^2 x + \sin^2 x \cos 2x = 0$

$\rightarrow \sin^2 x = 0$

$\sin^2 x (1 + \cos 2x) = 0 \rightarrow \cos 2x = -1$

$\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi + 0, \pi, 2\pi$

$\cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow x = \frac{2k\pi + \pi}{2} \rightarrow \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

پاسخ

سوال ۹ $\tan(45 - x) = \frac{\tan 45 - \tan x}{1 + \tan 45 \tan x} = \frac{1 - \tan x}{1 + \tan x}$

$\tan 2x = \tan(45 - x) \rightarrow 2x = k\pi + \frac{\pi}{4} - x \rightarrow 3x = k\pi + \frac{\pi}{4}$

$x = \frac{k\pi}{3} + \frac{\pi}{12}$

سوال ۱۰

$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x$

$\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x} = \sqrt{2} \cos 2x \rightarrow |\sin x + \cos x| = \sqrt{2} \cos 2x$

$\sin x + \cos x \geq 0 \rightarrow 0 \leq x \leq \frac{3\pi}{4}$ $\sqrt{2} \sin(x + \frac{\pi}{4}) = \sqrt{2} \cos 2x$

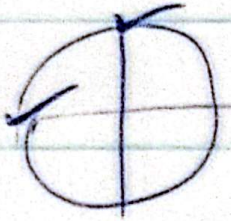
$\cos 2x = \cos(\frac{\pi}{4} - x) \rightarrow 2x = 2k\pi + x - \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{4}$ $\rightarrow \frac{7\pi}{4}$

$2x = 2k\pi + \pi - x + \frac{\pi}{4} \rightarrow 3x = 2k\pi + \frac{5\pi}{4} \rightarrow x = \frac{2k\pi}{3} + \frac{5\pi}{12}$ $\rightarrow \frac{5\pi}{12}$

تردجواب

سوال ۱۰ ...

$\sin^4 - \cos^4 = 1$  $\rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin^2 - \cos^2 = 1$  $\rightarrow 0, \pi$

خسته نباشین