۱۹/۲۵ آفرین خیلی خوب نوشتی

سوال ۱)

الف) $\cos 2\alpha - 3\sin\alpha - 1 = 0 \Rightarrow 1 - 2\sin^2\alpha = 3\sin\alpha - 1$

$2\sin^2\alpha + 3\sin\alpha - 2 = 0 \Rightarrow \sin^2\alpha + 3\sin\alpha - 4 = 0 \quad (\sin\alpha - 1)(\sin\alpha + 4)$ غ‌ق‌ق

$\sin\alpha = \frac{1}{2} \Rightarrow \alpha = 2k\pi + \frac{\pi}{6}$, $\alpha = 2k\pi + \pi - \frac{\pi}{6}$

ب) $\cos 2\alpha + \cos\alpha - 2 = 0 \Rightarrow 2\cos^2\alpha - 1 + \cos\alpha - 2 = 0 \Rightarrow$

$2\cos^2\alpha + \cos\alpha - 3 = 0 \Rightarrow \cos^2\alpha + \cos\alpha - 6 = 0 \Rightarrow (\cos\alpha + 3)(\cos\alpha - 2)$

غ‌ق‌ق $-\frac{3}{2}$ $\quad 1$

$\cos\alpha = 1 \Rightarrow \alpha = 2k\pi$

سوال ۲)

الف) $\sin^4\alpha - \cos^4\alpha = \sin 4\alpha \Rightarrow (\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha) = \sin 4\alpha$

$-\cos 2\alpha = \sin 4\alpha \Rightarrow \sin(-4\alpha) = \cos 2\alpha \Rightarrow \cos(\frac{\pi}{2} + 4\alpha) = \cos 2\alpha$

$\frac{\pi}{2} + 4\alpha = 2k\pi + 2\alpha \Rightarrow 2\alpha = 2k\pi - \frac{\pi}{2} \Rightarrow \alpha = k\pi - \frac{\pi}{4}$

$\frac{\pi}{2} + 4\alpha = 2k\pi - 2\alpha \Rightarrow 6\alpha = 2k\pi - \frac{\pi}{2} \Rightarrow \alpha = \frac{k\pi}{3} - \frac{\pi}{12}$

ب) $2\cos^2\alpha + 2\sin\alpha\cos\alpha = 1 \Rightarrow 2\sin\alpha\cos\alpha = 1 - 2\cos^2\alpha \Rightarrow \sin 2\alpha = -\cos 2\alpha$

$\Rightarrow \cos(\frac{\pi}{2} - 2\alpha) = -\cos 2\alpha \Rightarrow \cos(2\alpha - \frac{\pi}{2}) = \cos 2\alpha \Rightarrow 2\alpha - \frac{\pi}{2} = 2k\pi + 2\alpha$ غ‌ق‌ق

$2\alpha - \frac{\pi}{2} = 2k\pi - 2\alpha \Rightarrow 4\alpha = 2k\pi + \frac{\pi}{2} \Rightarrow \alpha = \frac{k\pi}{2} + \frac{\pi}{8}$

سوال ۳)

الف) $\frac{\cos\alpha}{\sin\alpha}(2\cos\alpha + 1) = \frac{1}{\sin\alpha} \Rightarrow \frac{2\cos^2\alpha + \cos\alpha}{\sin\alpha} = \frac{1}{\sin\alpha} \xrightarrow{\sin\alpha \neq 0}$

$2\cos^2\alpha + \cos\alpha - 1 = 0 \Rightarrow \cos^2\alpha + \cos\alpha - 2 = 0 \quad ((\cos\alpha - 1)(\cos\alpha + 2)$

$\frac{1}{2} \quad -1$

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$\cos\alpha = -1 \rightarrow \alpha = 2k\pi + \pi \rightarrow \sin = 0 \rightarrow$ حذف شده

$\cos\alpha = \frac{1}{2} \rightarrow \boxed{\alpha = 2k\pi + \frac{\pi}{3} , \alpha = 2k\pi - \frac{\pi}{3}}$

ب) $2\sin^2\alpha - \sin\alpha\cos\alpha + \cos^2\alpha = 2$

$2(1-\cos^2\alpha) + \cos^2\alpha - \sin\alpha\cos\alpha = 2$

$\cancel{2} - 2\cos^2\alpha + \cos^2\alpha - \sin\alpha\cos\alpha = \cancel{2}$

$-\cos^2\alpha - \sin\alpha\cos\alpha = 0 \Rightarrow \cos\alpha(\cos + \sin\alpha) = 0$

$\cos\alpha = 0 \rightarrow \boxed{\alpha = 2k\pi + \frac{\pi}{2} , \alpha = 2k\pi - \frac{\pi}{2}}$

$\sin(-\alpha) = \cos\alpha \rightarrow \cos\alpha = \cos(\frac{\pi}{2}+\alpha) \Rightarrow \frac{\pi}{2}+\alpha = 2k\pi + \alpha$ غیق

$\frac{\pi}{2}+\alpha = 2k\pi - \alpha \rightarrow 2\alpha = 2k\pi - \frac{\pi}{2} \rightarrow \boxed{\alpha = k\pi - \frac{\pi}{4}}$

سوال ۳

الف) $\cos(\frac{\pi}{4}+x)\cos(\pi - \frac{\pi}{4}) = \frac{1}{4} \rightarrow \frac{\pi}{4}+x+\frac{\pi}{4}-x = \frac{\pi}{2}$

$\cos(\frac{\pi}{4}+x)\cos(\frac{\pi}{4}-x) = \cos(\frac{\pi}{4}+x)\sin(\frac{\pi}{4}+x) = \frac{1}{2}\sin(\frac{\pi}{2}+2x) =$

$\frac{1}{2}\cos 2x = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{3}$

$\rightarrow \boxed{x = k\pi + \frac{\pi}{6}}$

$\rightarrow 2x = 2k\pi - \frac{\pi}{3} \rightarrow \boxed{x = k\pi - \frac{\pi}{6}}$

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$\cos(2x - \frac{\pi}{5}) = -\sin 2x \Rightarrow \sin(\frac{11\pi}{10} - 2x) = \sin(-2x)$

$3x = 2k\pi + x - \frac{11\pi}{18}$ غ ق

$\sin(3x) = \sin(x - \frac{11\pi}{18}) \xrightarrow{} 3x = 2k\pi + \pi - x + \frac{11\pi}{18}$

$4x = 2k\pi + \pi + \frac{11\pi}{18} \longrightarrow x = \frac{k\pi}{2} + \frac{\pi}{4} + \frac{11\pi}{72}$

سوال ۴

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \sin 2x \neq 0$

$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \cos^2 x + \sin^2 x \rightarrow 2\cos 2x + \cancel{1} = \cancel{1}$

②

$\cos 2\alpha = 0 \longrightarrow 2\alpha = 2k\pi + \frac{\pi}{2} \rightarrow \alpha = k\pi + \frac{\pi}{4}$

$2\alpha = 2k\pi - \frac{\pi}{2} \rightarrow \alpha = k\pi - \frac{\pi}{4}$

سوال ۵

$\sin\frac{x}{2} \neq 0 \,,\, \cos\frac{x}{2} \neq -1$

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} - \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \xrightarrow{\frac{x}{2}=\alpha} \frac{\sin\alpha}{1+\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$

②

$\sin^2\alpha = (1+\cos\alpha)^2 \rightarrow \cancel{1} - \cos^2\alpha = \cancel{1} + \cos^2\alpha + 2\cos\alpha$

$2\cos^2\alpha + 2\cos\alpha = 0 \longrightarrow 2\cos(\cos\alpha + 1) = 0$ → $\cos\alpha = 0$; → $\cos\alpha = -1$ غ ق

$\cos\frac{x}{2} = 0 \longrightarrow \frac{x}{2} = 2k\pi + \frac{\pi}{2} \rightarrow x = 4k\pi + \pi \rightarrow \pi$

$\frac{x}{2} = 2k\pi - \frac{\pi}{2} \rightarrow x = 4k\pi - \pi \rightarrow -\pi$

$|\pi - -\pi| = 2\pi$

سوال ۷

$\cos^2 x - \sin^2 x \cos 3x = 1$

$\cancel{1} - \sin^2 x - \sin^2 x \cos 3x = \cancel{1} \to \sin^2 x + \sin^2 x \cos 3x = 0$

$\to \sin^2 x = 0$ ب

$\sin^2 x (1+\cos 3x) = 0 \to \cos 3x = -1$

$\sin^2 x = 0 \to \sin x = 0 \to x = k\pi \to 0, \pi, 2\pi$

$\cos 3x = -1 \to 3x = 2k\pi + \pi \to x = \frac{2k\pi + \pi}{3} \to \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

۵ جواب

سوال ۸

$\tan(45 - x) = \frac{\tan 45 - \tan x}{1 + \tan 45 \tan x} = \frac{1 - \tan x}{1 + \tan x}$ ب

$\tan 3x = \tan(\underbrace{45 - x}_{\frac{\pi}{4}}) \to 3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4}$

$\boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$

سوال ۹

$\sqrt{1 + \sin 2x} = \sqrt{2} \cos 2x$ الف

$\sqrt{\sin^2 + \cos^2 + 2\sin x \cos x} = \sqrt{2} \cos 2x \to |\sin x + \cos x| = \sqrt{2} \cos 2x$

$0 \le x \le \frac{3\pi}{4} \xrightarrow{\sin + \cos \ge 0} \sqrt{2} \sin(x + \frac{\pi}{4}) = \sqrt{2} \cos 2x$

$\cos 2x = \cos(\underbrace{\frac{\pi}{4} - x}_{x - \frac{\pi}{4}}) \to 2x = 2k\pi + x - \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{4} \to \boxed{\frac{7\pi}{4}}$

$2x = 2k\pi + \pi - x + \frac{\pi}{4} \to 3x = 2k\pi + \frac{5\pi}{4} \to x \to \frac{2k\pi}{3} + \frac{5\pi}{12}$ $\cancel{\frac{5\pi}{12}}$

دو جواب

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود که این جواب را [illegible]

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سوال ۱

$\sin^4 - \cos^4 = 1$  $\rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

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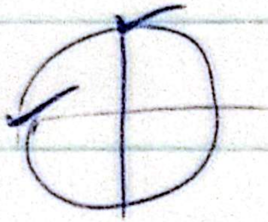
$\sin^3 - \cos^2 = -1$  $\rightarrow \frac{\pi}{2}, \pi$

صورت سوال را با دقت بنویس !!

$x = 0, 2\pi, \frac{3\pi}{2}$

خسته نباشی

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2-2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0 \to \begin{cases} \sin 2x = -1 \\ \sin 2x = +\frac{1}{2} \end{cases}$$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4}$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \ \text{یا}\ 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12} \ \text{یا}\ \frac{5\pi}{12}$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x=\frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

* معادله ۲ جواب دارد