

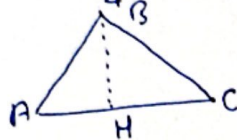
$$B: (0, \tan(-2x + \frac{\pi}{4})) \xrightarrow{x=0} \tan \frac{\pi}{4} = 1 \quad B|_0$$

$$C \rightarrow -2x + \frac{\pi}{4} = -\frac{\pi}{2} \rightarrow -2x = -\frac{3\pi}{4} \rightarrow x = +\frac{3\pi}{8}$$

$$A \rightarrow -2x + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow -2x = \frac{\pi}{4} \rightarrow x = -\frac{\pi}{8}$$

$$AC = \frac{\pi}{2}$$

$$BH = 1$$



$$\Rightarrow S = \frac{1}{2} AC \times BH = \frac{\pi}{4}$$

1

$$x^2 - (\sin \alpha)x - \frac{1}{4} \cos^2 \alpha = 0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9} - \frac{2}{3} \sin \alpha - \frac{1}{4} \cos^2 \alpha = 0 \xrightarrow{\cos^2 \alpha = 1 - \sin^2 \alpha}$$

$$-\frac{1}{4}(1 - \sin^2 \alpha) - \frac{2}{3} \sin \alpha + \frac{4}{9} \xrightarrow{\times 36} -9 + 9 \sin^2 \alpha - 24 \sin \alpha + 16 \rightarrow 9 \sin^2 \alpha - 24 \sin \alpha + 7$$

$$\rightarrow \Delta = 324 \quad \sin \alpha = \frac{24 \pm 18}{18} \rightarrow \frac{42}{18} \text{ و } \frac{6}{18}$$

$$1 - \sin^2 \alpha = \cos^2 \alpha \rightarrow \frac{8}{9} \quad \frac{6}{18} = \frac{1}{3} \text{ ن.ق}$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} = \frac{9}{8}$$

2

$$\log(\sin \alpha) - \log(-\cos \alpha) = \log 3 \rightarrow \log \frac{\sin \alpha}{-\cos \alpha} = \log -\tan \alpha = \log 3 \rightarrow \tan \alpha = -3$$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} \rightarrow \frac{1}{\cos^2 \alpha} = 10 \rightarrow \cos \alpha = \pm \frac{\sqrt{10}}{10} \xrightarrow{\log \tan \alpha} \cos \alpha = -\frac{\sqrt{10}}{10}$$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = \frac{9}{10} \rightarrow \frac{3\sqrt{10}}{10} \xrightarrow{\log \tan \alpha} \sin \alpha = +\frac{3\sqrt{10}}{10}$$

$$\sin \alpha - \cos \alpha = \frac{3\sqrt{10}}{10} - (-\frac{\sqrt{10}}{10}) = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$$

3

$$\sin^4 \theta + \cos^2 \theta = \frac{4}{5} \rightarrow (\sin^4 \theta) = (\sin^2 \theta)^2 = (1 - \cos^2 \theta)^2 = \cos^4 \theta - 2\cos^2 \theta + 1$$

$$\cos^4 \theta - \cos^2 \theta + 1 = \frac{4}{5}$$

عبارت خواسته شده را می توانیم بنویسیم

$$\cos^4 \theta + \sin^2 \theta \rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^4 \theta - \cos^2 \theta + 1$$

4

$$\frac{3 \sin x + \sin x \cos x}{2 - 2 \cos^2 x} < 0 \rightarrow \frac{\sin x (3 + \cos x)}{2(1 - \cos^2 x)} < 0 \Rightarrow \frac{3 + \cos x}{2 \sin x} < 0 \rightarrow -1 < \cos x < 1$$

چون $\cos x < 1$ و $\sin x < 0$ و $\cos x + 3 > 0$ باشد.

$$\sin x \tan x - \frac{1}{\cos x} > 0 \rightarrow \frac{\sin^2 x - 1}{\cos x} > 0 \Rightarrow \frac{1 - \cos^2 x - 1}{\cos x} > 0 \Rightarrow -\cos x > 0 \rightarrow \cos x < 0$$

در نتیجه در ناحیه چهارم می باشد.

5

$$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \rightarrow \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \rightarrow 2\cos x + 3\sin x = 0 \rightarrow 2\cos x = -3\sin x$$

$$\cos x = -\frac{3}{2}\sin x \rightarrow \tan x = \frac{\sin x}{\cos x} = \frac{\sin x}{-\frac{3}{2}\sin x} = -\frac{2}{3}, \cot x = \frac{1}{\tan x} = -\frac{3}{2}$$

$$\tan x + \cot x = -\frac{3}{2} - \frac{2}{3} = \frac{-9-4}{6} = -\frac{13}{6}$$

8

$$1 + \sin 50^\circ = (\sin 25^\circ + \cos 25^\circ)^2 \rightarrow \sqrt{1 + \sin 50^\circ} = \sin 25^\circ + \cos 25^\circ = \frac{1}{\sqrt{2}} \sqrt{1+1} \sin(25^\circ + 45^\circ)$$

$$\sqrt{2} \sin 70^\circ \xrightarrow{70^\circ + 20^\circ = 90^\circ} = \sqrt{2} \cos 20^\circ$$

$$\sin 50^\circ + \sin 10^\circ \Rightarrow \sin(30^\circ + 20^\circ) + \sin(30^\circ - 20^\circ) = \sin 30^\circ \cos 20^\circ + \sin 20^\circ \cos 30^\circ + \sin 30^\circ \cos 20^\circ - \sin 20^\circ \cos 30^\circ$$

$$= 2\sin 30^\circ \cos 20^\circ \Rightarrow \cos 20^\circ$$

$$\frac{\sqrt{2} \cos 20^\circ}{\cos 20^\circ} = \sqrt{2}$$

9

$$(1 - 2\sin^2 5^\circ)(2\cos^2 10^\circ - 1) \left(\frac{1 - \tan^2 20^\circ}{1 + \tan^2 20^\circ} \right) = \cos 10^\circ \times \cos 20^\circ \times \cos 40^\circ \times \frac{\sin 10^\circ}{\sin 10^\circ}$$

$$1 - 1 + \cos 10^\circ \quad 1 + \cos 20^\circ - 1 \quad \cos 40^\circ \quad \frac{1}{4} \sin 40^\circ$$

$$\frac{1}{2} \sin 20^\circ \sin 10^\circ \cos 10^\circ \cos 20^\circ \cos 40^\circ = \frac{1}{2} \sin 20^\circ \cos 20^\circ \cos 40^\circ \frac{1}{4} \sin 40^\circ \cos 40^\circ = \frac{1}{8} \frac{\sin 80^\circ}{\sin 10^\circ}$$

$$10^\circ + 80^\circ = \frac{\pi}{2} \rightarrow \frac{1}{8} \frac{\cos 10^\circ}{\sin 10^\circ} = \frac{1}{8} \cot 10^\circ$$

10

$$\sin \alpha + 3\cos \alpha = 2 \xrightarrow{\wedge^2} \sin^2 \alpha + 9\cos^2 \alpha + 6\cos \alpha \sin \alpha = 4 \xrightarrow{\sin^2 \alpha + \cos^2 \alpha = 1} 8\cos^2 \alpha + 6\cos \alpha \sin \alpha = 3$$

$$\rightarrow 1 - 2\cos^2 \alpha = \cos 2\alpha \rightarrow 1 - \cos 2\alpha = 2\cos^2 \alpha, \sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\rightarrow 4 - 4\cos 2\alpha + 3\sin 2\alpha = 3$$

9

$$A + B + C = 180^\circ \rightarrow C = 108^\circ - A \quad \cos^2 \frac{A}{2} = \frac{\cos A + 1}{2} = \sin 72^\circ \times \sin 108^\circ - A$$

$$\frac{\cos A + 1}{2} = \sin 72^\circ \times \sin C$$

$$\cos A = -\cos -A = \cos(B+C) = \cos B \cos C - \sin B \sin C$$

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