حسابان / تکلیف شماره ی ۱۵

①

$y=\tan(-2x+\frac{\pi}{4})$ $\quad T=\frac{\pi}{|b|}=\frac{\pi}{2} \rightarrow x=0 \quad y=1$

$S=\frac{\frac{\pi}{2}\times 1}{2}=\frac{\pi}{4}$

②

$x^2-(\sin\alpha)x-\frac{1}{4}\overbrace{\cos^2\alpha}^{1-\sin^2\alpha}=0 \rightarrow x^2-(\sin\alpha)x-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0 \rightarrow$

$\xrightarrow{x=\frac{2}{3}} \frac{4}{9}-(\sin\alpha)\frac{2}{3}-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0 \longrightarrow$

$\frac{1}{4}\sin^2\alpha-\frac{2}{3}(\sin\alpha)+\frac{7}{36}=0 \xrightarrow{\times 4} \sin^2\alpha-\frac{8}{3}\sin\alpha+\frac{7}{9}=0$

$\xrightarrow{\sin\alpha=x} (x-\frac{1}{3})(x-\frac{7}{3})=0 \begin{cases} \rightarrow \sin\alpha=\frac{1}{3} \ ✓ \\ \rightarrow \sin\alpha=\frac{7}{3} \ ✗ \end{cases}$

$\cos^2\alpha=1-\frac{1}{9}=\frac{8}{9}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{9}{8}$

③

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \qquad \sin\alpha - \cos\alpha = ?$

$\log\frac{-\sin\alpha}{\cos\alpha} = \log(-\tan\alpha) = \log 3 \rightarrow -\tan\alpha = 3 \rightarrow \tan\alpha = -3$

$\sin\alpha > 0 \quad , \quad -\cos\alpha > 0 \rightarrow \cos\alpha < 0$

$1+\underbrace{\tan^2\alpha}_{9} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{1}{10} \rightarrow \cos\alpha = \frac{-1}{\sqrt{10}} \qquad \sin\alpha = \frac{3}{\sqrt{10}}$

$\sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$

④

$\sin^4\theta + \underbrace{\cos^4\theta}_{(1-\sin^2\theta)^2} = \frac{4}{5} \qquad \cos^4\theta + \sin^2\theta = ?$

$\rightarrow \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$

$(1-\sin^2\theta)^2 + \sin^2\theta = \sin^4\theta - 2\sin^2\theta + 1 + \sin^2\theta =$

$\sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$

$2 \le 3+\cos x \le 4 \to$ همواره $\oplus$

⑤

$$\frac{3\sin x+\sin x\cos x}{2-2\cos^2 x}<0 \to \frac{\sin x(3+\cos x)}{2\underbrace{(1-\cos^2 x)}_{\sin^2 x}}<0 \to \sin x<0$$

$$\sin x\tan x-\frac{1}{\cos x}>0 \to \frac{\sin^2 x-1}{\cos x}>0 \to$$

$$\frac{-\cos^2 x}{\cos x}>0 \to -\cos x>0 \to \cos x<0$$

$\sin x<0 \, , \cos x<0 \longrightarrow$ ناحیه $\underline{3}$

⑥

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \qquad \tan x+\cot x=$$

$$\frac{2\cos x+3\sin x}{\sin x\cos x}=0 \to 2\cos x+3\sin x=0 \to$$

$$2\cos x=-3\sin x \to \tan x=-\frac{2}{3} \, , \cot x=\frac{-3}{2}$$

$$\tan x+\cot x=\frac{-2}{3}-\frac{3}{2}=\frac{-4-9}{6}=\frac{-13}{6}$$

⑦

$$\frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}=\frac{\sqrt{(\sin 25+\cos 25)^2}}{\sin 50+\sin 10}=\frac{\sqrt{2}\sin(25+45)}{\sin 50+\sin 10}$$

$$\frac{\sqrt{2}\sin 70}{\sin 50+\sin 10}$$

(۸)

$$\underbrace{(1-2\sin^2 5^\circ)}_{\cos 10}\ \underbrace{(2\cos^2 10-1)}_{\cos 20}\ \underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{\cos 40}$$

$$\frac{\sin 10\cos 10\cos 20\cos 40}{\sin 10}=\frac{\sin 20\cos 20\cos 40}{2\sin 10}=$$

$$\frac{\sin 40\cos 40}{4\sin 10}=\frac{\sin 80}{8\underbrace{\sin 10}_{\cos 80}}=\frac{\tan 80}{8}$$

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(۹)

$\sin\alpha+3\cos\alpha=2$ — $5\cos 2\alpha-12\cos\alpha$

$\Rightarrow \sin\alpha=2-3\cos\alpha$ — $\Rightarrow 5(2\cos^2\alpha-1)=10\cos^2\alpha-5-12\cos\alpha$

$\xrightarrow{\text{توان ۲}} \sin^2\alpha+6\sin\alpha\cos\alpha+9\cos^2\alpha=4$

$1-\cos^2\alpha+9\cos^2\alpha+\underbrace{6\cos\alpha(2-3\cos\alpha)}_{12\cos\alpha-18\cos^2\alpha}=3 \rightarrow$

$-10\cos^2\alpha+12\cos\alpha=3 \rightarrow$

$\underbrace{10\cos^2\alpha-12\cos\alpha}_{-3}-5=-8$

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(۱۰)

$\sin\hat{B}\sin\hat{C}=\underbrace{\cos^2\frac{A}{2}}_{\frac{1+\cos A}{2}} \rightarrow 2\sin\hat{B}\sin\hat{C}=1+\cos A$ — $\hat{B}=72^\circ$