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مسابقاتی / تکلیف شماره‌ی ۱۵

(۱)

$y = \tan(-2x + \frac{\pi}{4})$ $\quad T = \frac{\pi}{|b|} = \frac{\pi}{2} \rightarrow x = 0 \quad y = 1$

(۲)

$S = \frac{\frac{\pi}{2} \times 1}{2} = \frac{\pi}{4}$ ✓

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(۴)

$x^2 - (\sin\alpha)x - \frac{1}{4}\underbrace{\cos^2\alpha}_{1-\sin^2\alpha} = 0 \rightarrow x^2 - (\sin\alpha)x - \frac{1}{4} + \frac{1}{4}\sin^2\alpha = 0 \rightarrow$

$\xrightarrow{x = \frac{2}{3}} \frac{4}{9} - (\sin\alpha)\frac{2}{3} - \frac{1}{4} + \frac{1}{4}\sin^2\alpha = 0 \longrightarrow$

(۲)

$\frac{1}{4}\sin^2\alpha - \frac{2}{3}(\sin\alpha) + \frac{7}{36} = 0 \xrightarrow{\times 4} \sin^2\alpha - \frac{8}{3}\sin\alpha + \frac{7}{9} = 0$

$\xrightarrow{\sin\alpha = x} (x - \frac{1}{3})(x - \frac{7}{3}) = 0 \begin{cases} \rightarrow \sin\alpha = \frac{1}{3} ✓ \\ \rightarrow \sin\alpha = \frac{7}{3} ✗ \end{cases}$

$\cos^2\alpha = 1 - \frac{1}{9} = \frac{8}{9}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{9}{8}$ ✓

(۳)
$\log^{(\sin\alpha)} - \log^{(-\cos\alpha)} = \log^{۳} \quad \sin\alpha - \cos\alpha = ?$

$\log^{\frac{-\sin\alpha}{\cos\alpha}} = \log^{-\tan\alpha} = \log^{۳} \rightarrow -\tan\alpha = ۳ \rightarrow \tan\alpha = -۳$

(۲)

$\sin\alpha > 0 \quad , \quad -\cos\alpha > 0 \rightarrow \cos\alpha < 0$

$1 + \tan^{۲}\alpha = \frac{1}{\cos^{۲}\alpha} \rightarrow \cos^{۲}\alpha = \frac{1}{10} \rightarrow \cos\alpha = \frac{-1}{\sqrt{10}} \quad \sin\alpha = \frac{۳}{\sqrt{10}}$
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$\sin\alpha - \cos\alpha = \frac{۳}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{۴}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{۴\sqrt{10}}{10} = \frac{۲\sqrt{10}}{۵}$

(۴)
$\sin^{۴}\theta + \cos^{۴}\theta = \frac{۴}{۵} \quad \cos^{۴}\theta + \sin^{۲}\theta = ?$

$1 - \sin^{۲}\theta \hookrightarrow \sin^{۴}\theta - \sin^{۲}\theta + 1 = \frac{۴}{۵}$

(۲)

$(1 - \sin^{۲}\theta)^{۲} + \sin^{۲}\theta = \sin^{۴}\theta - ۲\sin^{۲}\theta + 1 + \sin^{۲}\theta =$

$\sin^{۴}\theta - \sin^{۲}\theta + 1 = \frac{۴}{۵}$

$2 < 3 + \cos x < 4 \to$ همواره (+)

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$$\frac{3\sin x + \sin x\cos x}{2-2\cos^2 x} < 0 \to \frac{\sin x(3+\cos x)}{2\underbrace{(1-\cos^2 x)}_{\sin^2 x}} < 0 \to \sin x < 0$$

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$$\sin x \tan x - \frac{1}{\cos x} > 0 \to \frac{\sin^2 x - 1}{\cos x} > 0 \to$$

$$\frac{-\cos^2 x}{\cos x} > 0 \to -\cos x > 0 \to \cos x < 0$$

$\sin x < 0 \ , \ \cos x < 0 \longrightarrow$ ناحیه ۳

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$$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \qquad \tan x + \cot x$$

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$$\frac{2\cos x + 3\sin x}{\sin x\cos x} = 0 \to 2\cos x + 3\sin x = 0 \to$$

$$2\cos x = -3\sin x \to \tan x = \frac{-2}{3} \ , \ \cot x = \frac{-3}{2}$$

$$\tan x + \cot x = \frac{-2}{3} - \frac{3}{2} = \frac{-4-9}{6} = \frac{-13}{6}$$

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۱,۲۵

$$\frac{\sqrt{1+\sin 50}}{\sin 50 + \sin 10} = \frac{\sqrt{(\sin 25 + \cos 25)^2}}{\sin 50 + \sin 10} = \frac{\sqrt{2}\sin(25+45)}{\sin 50 + \sin 10}$$

$$\frac{\sqrt{2}\sin 70}{\sin 50 + \sin 10}$$

$$\sqrt{1+\sin 50^\circ} = \sqrt{1+\cos 40^\circ} \xrightarrow[1+\cos 40^\circ = 2\cos^2 20^\circ]{\text{فرمول طلایی}} \sqrt{2\cos^2 20^\circ} = \sqrt{2}|\cos 20^\circ| = \sqrt{2}\cos 20^\circ$$

$$\sin 50^\circ + \sin 10^\circ = \sin(30^\circ + 20^\circ) + \sin(30^\circ - 20^\circ) =$$

$$\sin 30^\circ\cos 20^\circ + \cancel{\sin 20^\circ\cos 30^\circ} + \sin 30^\circ\cos 20^\circ - \cancel{\sin 20^\circ\cos 30^\circ} = 2\left(\frac{1}{2}\right)\cos 20^\circ = \cos 20^\circ$$

$$\frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \boxed{\sqrt{2}}$$

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۲٫۷۵

$$\underbrace{(1-2\sin^2 5)}_{\cos 10}\;\underbrace{(2\cos^2 10-1)}_{\cos 20}\;\left(\underbrace{\frac{1-\tan^2 20}{1+\tan^2 20}}_{\cos 40}\right)$$

$$\frac{\sin 10\cos 10\cos 20\cos 40}{\sin 10}=\frac{\sin 20\cos 20\cos 40}{2\sin 10}=$$

$$\frac{\sin 40\cos 40}{4\sin 10}=\frac{\sin 80}{8\underbrace{\sin 10}_{\cos 80}}=\frac{\tan 80}{8}=\frac{1}{8}\cot 10^\circ$$

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$\sin\alpha+3\cos\alpha=2$ $\qquad 5\cos 2\alpha-12\cos\alpha$

$\hookrightarrow \sin\alpha=2-3\cos\alpha$ $\qquad \hookrightarrow 5(2\cos^2\alpha-1)=10\cos^2\alpha-5-12\cos\alpha$

$\xrightarrow{\text{توان ۲}} \sin^2\alpha+6\sin\alpha\cos\alpha+9\cos^2\alpha=4$

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$$1-\cos^2\alpha+9\cos^2\alpha+\underbrace{6\cos\alpha(2-3\cos\alpha)}_{12\cos\alpha-18\cos^2\alpha}=3\rightarrow$$

$-10\cos^2\alpha+12\cos\alpha=3\rightarrow$

$\underbrace{10\cos^2\alpha-12\cos\alpha}_{-3}-5=-8$

۱۰

$\sin\hat B\sin\hat C=\cos^2\frac{A}{2}\rightarrow 2\sin\hat B\sin\hat C=1+\cos A$ $\qquad \hat B=72^\circ$

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$$\cos^2\frac{A}{2}=\frac{1+\cos A}{2}\xrightarrow{} 2\sin B\sin C=1+\cos\hat A\xrightarrow{A+B+C=\pi}A=\pi-(\hat B+\hat C)$$

$1+\cos(\pi-(B+C))=1-\cos(B+C)=2\sin B\sin C$

$1-\cos B\cos C+\sin B\sin C=2\sin B\sin C\rightarrow \sin B\sin C+\cos B\cos C=1$

$\cos(B-C)=1\rightarrow \begin{cases}B-C=0\\ B-C=2\pi \;\times \text{ مثلث}\end{cases}\rightarrow B=C\rightarrow A=180^\circ-144^\circ=36^\circ$

$B+C=2B \quad (2\times 72)$