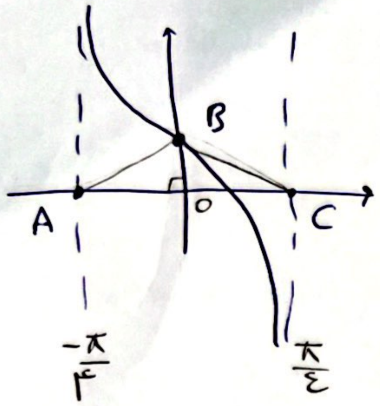
بنام خداوندان رحیم

تالیف نگاره لیا



(۱)

$$y = \tan\left(-2x + \frac{\pi}{4}\right)$$

$S_{\overset{\triangle}{ABC}} = ?$ $\quad x = 0 \Rightarrow y = \tan\frac{\pi}{4} = 1 \Rightarrow$ $B = 1$

دوره تناوب

$$T = \frac{\pi}{|a|} = \frac{\pi}{|-2|} = \frac{\pi}{2}$$

a ضریب

$$S_{ABC} = \frac{BO \times AC}{2} = \frac{1 \times \frac{\pi}{2}}{2} = \frac{\pi}{4}$$

---

(۲)

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \longrightarrow \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}(1 - \sin^2\alpha) = 0$

$x = \frac{2}{3} \longrightarrow y = 0$

$1 + \tan^2\alpha = ?$

$\frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \frac{4}{9} - \frac{1}{4} = 0 \xrightarrow{\times 36}$

$\frac{16-9}{36} = \frac{7}{36}$

$9\sin^2\alpha - 24\sin\alpha + 7 = 0$

$\sin^2\alpha - 24\sin\alpha + 63 = 0 \Rightarrow$

$(\sin\alpha - 3)(\sin\alpha - 21) = 0 \xrightarrow{\div 9} \sin\alpha = \frac{3}{9} = \frac{1}{3}$

$\longrightarrow \sin\alpha = \frac{21}{9} = \frac{7}{3}$ غق

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{1 - \sin^2\alpha} =$$

$$\frac{1}{1 - \frac{1}{9}} = \frac{1}{\frac{8}{9}} = \frac{9}{8}$$

---

(۳)

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$ $\qquad \sin\alpha - \cos\alpha = ?$

$\log\frac{\sin\alpha}{-\cos\alpha} = \log 3 \Rightarrow \frac{\sin\alpha}{-\cos\alpha} = 3 \longrightarrow -3\cos\alpha = \sin\alpha \Rightarrow \sin^2\alpha = 9\cos^2\alpha$

$\sin^2 + \cos^2 = 1 \Rightarrow 10\cos^2\alpha = 1 \longrightarrow \cos^2\alpha = \frac{1}{10}$

$\Rightarrow \cos\alpha = \frac{-\sqrt{10}}{10}$

$\Rightarrow \sin^2\alpha = \frac{9}{10} \Rightarrow \sin\alpha = \frac{3\sqrt{10}}{10}$

$\Rightarrow \sin\alpha - \cos\alpha = \frac{3\sqrt{10}}{10} + \frac{\sqrt{10}}{10} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$

شرط log

$\sin\alpha > 0$

$-\cos\alpha > 0$

$\cos\alpha < 0$

$\Downarrow$

ناحیه دوم

(٤)

$$\sin^4\theta + \cos^2\theta = \frac{4}{5}$$

$$\cos^4\theta + \sin^2\theta = ? \longrightarrow \cos^4\theta + \sin^2\theta = (\cos^2\theta)^2 + \sin^2\theta =$$

$$(1-\sin^2\theta)^2 + \sin^2\theta = \sin^4\theta - 2\sin^2\theta + 1 + \sin^2\theta = \sin^4\theta - \sin^2\theta + 1$$

$$\Rightarrow \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}$$

باهم برابرند

$$\sin^4\theta + 1 - \sin^2\theta = \frac{4}{5}$$

---

(٥)

$$\frac{3\sin x + \sin x \cos x}{\underbrace{2-2\cos^2 x}} < 0 \longrightarrow \frac{\sin x\,(3+\cos x)}{2(1-\underbrace{\cos^2 x}_{\sin^2 x})} < 0 \Rightarrow$$

همواره مثبت

$$\frac{(3+\cos x)}{2\sin x} < 0 \longrightarrow \sin x < 0$$

$$\sin x \tan x - \frac{1}{\cos x} > 0 \qquad \frac{\sin^2 x - 1}{\cos x} > 0 \rightarrow \frac{-\cos^2 x}{\cos x} > 0 \Rightarrow \cos x < 0$$

ناحیه سوم

---

(٦)

$$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \Rightarrow \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \Rightarrow 3\sin x = -2\cos x$$

$$\tan x + \cot x = ? \qquad \Longrightarrow \sin x = -\frac{2}{3}\cos x \Rightarrow$$

sin و cos مختلف العلامت اند

$$\tan x + \cot x = \frac{1}{\sin x \cdot \cos x} = \frac{1}{-\frac{3}{\sqrt{13}} \times \frac{2}{\sqrt{13}}} =$$

$$\cos^2 x + \frac{4}{9}\cos^2 x = 1 \Rightarrow \frac{13}{9}\cos^2 x = 1$$

$$\Rightarrow \cos^2 x = \frac{9}{13} \Rightarrow \cos x = \pm\frac{3}{\sqrt{13}}$$

$$\sin^2 x = \frac{4}{13} \Rightarrow \sin x = \pm\frac{2}{\sqrt{13}}$$

$$\frac{1}{\frac{-6}{13}} = \frac{-13}{6}$$

(7)

$$y=\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}\times\frac{\sqrt{1-\sin 50^\circ}}{\sqrt{1-\sin 50^\circ}}=\frac{\sqrt{1-\sin^2 50^\circ}}{\cos 20^\circ\sqrt{1-\sin 50}}=\frac{\cos 50 \rightarrow \sin 40^\circ}{\cos 20(\sqrt{1-\sin 50})}=$$

$$\sin 50^\circ+\sin 10^\circ=2\sin\frac{60^\circ}{2}\times\cos\frac{40}{2}=2\underbrace{\sin 30^\circ}_{\frac{1}{2}}\cos 20^\circ=\cos 20^\circ$$

$$\frac{2\sin 20^\circ\cos 20^\circ}{\cos 20^\circ\sqrt{1-\sin 50^\circ}}=$$

$$\frac{2\sin 20^\circ}{\sqrt{1-\sin 50^\circ}}=\frac{2\sin 20^\circ}{\sqrt{1-\cos 40}}=\frac{2\sin 20^\circ}{\sqrt{1-(1-2\sin^2 20^\circ)}}=\frac{2\sin 20^\circ}{\sqrt{2}\sin 20^\circ}=\sqrt{2}$$

ج آخر

---

(8)

$$(1-2\sin^2 5^\circ)(2\cos^2 10^\circ-1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)=\cos 10^\circ\cos 20^\circ\cos 40^\circ$$

$\cos 10^\circ$ $\quad$ $\cos 20^\circ$ $\quad$ $\cos 40^\circ$

$\times\sin 10^\circ$
$\div\sin 10^\circ$

$$\frac{\sin 10^\circ\cos 10^\circ\cos 20^\circ\cos 40^\circ}{\sin 10^\circ}=\frac{1}{2}\frac{\sin 20^\circ\cos 20^\circ\cos 40^\circ}{\sin 10^\circ}=\frac{1}{4}\frac{\sin 40^\circ\cos 40^\circ}{\sin 10^\circ}=$$

$$\frac{\frac{1}{8}\overset{\cos 10}{\sin 80^\circ}}{\sin 10}=\frac{1}{8}\cot 10^\circ$$

---

(9)

$$\sin\alpha+3\cos\alpha=2$$
$$5\cos 2\alpha-12\cos\alpha=?$$

$$\sin\alpha=2-3\cos\alpha\Rightarrow$$
$$\cos^2\alpha+(2-3\cos\alpha)^2=1\Rightarrow 9\cos^2\alpha-12\cos\alpha+4+\cos^2\alpha=1$$
$$\Rightarrow 10\cos^2\alpha-12\cos\alpha+3=0$$

$$\left.\begin{array}{l}5(2\cos^2\alpha-1)-12\cos\alpha=10\cos^2\alpha-5-12\cos\alpha\\ 10\cos^2\alpha-12\cos\alpha=-3\end{array}\right\}\Rightarrow$$

$$10\cos^2\alpha-12\cos\alpha-5=-3-5=-8$$

ج آخر

$\sin\hat{B} \cdot \sin\hat{C} = \cos^{۲}\frac{\hat{A}}{۲} \quad , \hat{B} = ۷۲^{\circ} \qquad \hat{A} = ?$ ۱۰

$$\cos^{۲}\frac{\hat{A}}{۲} = \frac{۱+\cos\hat{A}}{۲}$$

$$\left.\begin{aligned} \cos(B-C) &= \cos\hat{B}\cos\hat{C} + \sin\hat{B}\sin\hat{C} \\ \cos(B+C) &= \cos\hat{B}\cos\hat{C} - \sin\hat{B}\sin\hat{C} \end{aligned}\right\} \longrightarrow \frac{\cos(B-C) - \cos(B+C)}{۲} = \sin\hat{B} \cdot \sin\hat{C}$$

$$\Longrightarrow \cos(\hat{B}+\hat{C}) = \cos(۱۸۰ - \hat{A}) = -\cos\hat{A}$$

$$\frac{\cos(B-C) + \cos\hat{A}}{۲} = \frac{۱+\cos\hat{A}}{۲} \Longrightarrow \cos(\hat{B}-\hat{C}) = ۱ \overset{\cos ۰^{\circ} = ۱}{\Longrightarrow}$$

$$\hat{B} - \hat{C} = ۰ \Longrightarrow \hat{B} = \hat{C} = ۷۲^{\circ}$$

$$\hat{A} = ۱۸۰ - \left(\underset{۱۴۴}{۲\times۷۲}\right) = ۳۶^{\circ}$$