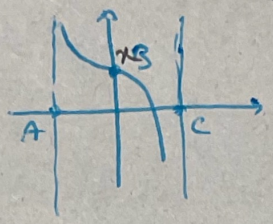
تدریس رندی

سؤال ۱

$y = \tan(-2x + \frac{\pi}{4})$

$S_{\triangle ABC} = ?$



$B \Big| \dot{x} \rightarrow \tan\frac{\pi}{4} = 1$

$S = \frac{1 \times \frac{\pi}{2}}{2} = \frac{\pi}{4}$ ✓

گزینه ۲

سؤال ۲

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$ $\quad 1 + \tan^2\alpha = ?$

$(\frac{2}{2})^2 - (\frac{2}{2}\sin\alpha) - \frac{1}{4}\cos^2\alpha = 0 \Rightarrow \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0$

$\frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha$ $\quad x = \frac{\sin\alpha \pm \sqrt{\sin^2\alpha + \cos^2\alpha}}{2} = \frac{\sin\alpha \pm 1}{2}$

$\frac{\sin\alpha \pm 1}{2} = \frac{2}{3}$

$\frac{\sin\alpha + 1}{2} = \frac{2}{3} \Rightarrow \sin\alpha = \frac{1}{3}$ ✓

$\frac{\sin\alpha - 1}{2} = \frac{2}{3} \Rightarrow \sin\alpha = \frac{5}{3}$ ق.ق.ق

$1 = \sin\alpha \Rightarrow \sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{\sqrt{8}}{3}$

$1 + \tan^2\alpha = 1 + \frac{\sin^2\alpha}{\cos^2\alpha} = 1 + \frac{(\frac{1}{3})^2}{(\frac{\sqrt{8}}{3})^2} = 1 + \frac{\frac{1}{9}}{\frac{8}{9}} = 1 + \frac{1}{8} = \frac{9}{8}$

گزینه ۲

سؤال ۳

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$ $\quad \sin\alpha - \cos\alpha = ?$

$\log a - \log b = \log(\frac{a}{b}) \Rightarrow \log(\frac{\sin\alpha}{-\cos\alpha}) = \log 3 \Rightarrow \frac{\sin\alpha}{-\cos\alpha} = 3$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = -3 \Rightarrow \tan\alpha = -3$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (-3\cos\alpha)^2 + \cos^2\alpha = 1 \Rightarrow 9\cos^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{1}{10}$

$\Rightarrow \cos\alpha = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}}$ $\quad \sin\alpha = -\frac{3}{\sqrt{10}}$ $\quad \Big\} \sin\alpha - \cos\alpha = -\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}} = -\frac{4}{\sqrt{10}}$

$\cos\alpha = -\frac{1}{\sqrt{10}} \Rightarrow \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}}$ ✓

سؤال ۴

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \Rightarrow \sin^2\theta + \cos^2\theta = 1$ $\quad x = \sin^2\theta$, $y = \cos^2\theta$

$\cos^4\theta + \sin^2\theta = ?$

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$$\begin{cases} x^2 + y = \frac{4}{5} \Rightarrow x^2 + (1-x) = \frac{4}{5} \Rightarrow x^2 - x = -\frac{1}{5} \\ x + y = 1 \Rightarrow y = 1 - x \end{cases}$$

$\cos^4\theta + \sin^2\theta = y^2 + x = (1-x)^2 + x = 1 + x^2 - 2x + x =$

$x + y = 1 \Rightarrow y = 1 - x$ $\qquad 1 + x^2 - x = \frac{4}{5}$

$\Rightarrow \cos^4\theta + \sin^2\theta = \frac{4}{5}$ ✓

سؤال ۹

$$\frac{2\sin x + \sin x \cos x}{2\underbrace{(1-\cos^2 x)}_{\sin^2 x}} < 0$$

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$$\frac{\sin(2 + \cos x)}{2\sin^2 x} < 0 \Rightarrow \sin < 0$$

$\sin x \left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) > 0$ $\quad$ ضرب در $\cos^2 x$

$\frac{\sin^2 x - 1}{\cos x} > 0$ $\quad \frac{-\cos^2 x}{\cos x} > 0 \Rightarrow \cos < 0$ ✓

سؤال ۹

$\frac{2}{\sin x} + \frac{2}{\cos x} = 0 \Rightarrow \frac{2}{\sin x} = -\frac{2}{\cos x} \xrightarrow{\times \sin x \cos x} 2\cos x = -2\sin x$

$\Rightarrow \frac{\cos x}{\sin x} = -\frac{2}{2} \Rightarrow \cot x = -\frac{2}{2} \Rightarrow \tan x = -\frac{2}{2} \Rightarrow -\frac{\pi}{2} - \frac{\pi}{4} = \boxed{-\frac{3\pi}{4}}$ ✓

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سؤال ۷

$\frac{\sqrt{1 + \sin 20}}{\sin 70 + \sin 10} = ?$ $\qquad$ ۱۵

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ $\quad A = 20,\ B = 10$

$\Rightarrow \sin 20 + \sin 10 = 2\cos 30$

سؤال ①

$$\underbrace{(1-2\sin^2\theta)}_{①}\underbrace{(2\cos^2 10-1)}_{②}\underbrace{\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)}_{③}$$

① $1-2\sin^2\theta=\cos 10$

② $2\cos^2 10-1=\cos 20$

③ $\frac{1-\tan^2 20}{1+\tan^2 20}=\cos 40$

$\cos 10\times\cos 20\times\cos 40$

$\cos 10\times 2\cos^2 10-1$

$\cos 40=2\cos^2 20-1=2(2\cos^2 10-1)^2-1$

$\Rightarrow$ درنهایت اصلی $\cos 10(2\cos^2 10-1)(2(2\cos^2 10-1)^2-1)$

سؤال ⑨

$\sin\alpha+2\cos\alpha=2$

$5\cos 2\alpha-12\cos\alpha=?$

$\cos 2\alpha=2\cos^2\alpha-1$

$5(2\cos^2\alpha-1)-12\cos\alpha=10\cos^2\alpha-12\cos\alpha-5$ (I)

$\sin\alpha+2\cos\alpha=2\Rightarrow\sin\alpha=2-2\cos\alpha\Rightarrow\sin^2\alpha=(2-2\cos\alpha)^2$

$\Rightarrow(2-2\cos\alpha)^2+\cos^2\alpha=1\Rightarrow 4-12\cos\alpha+9\cos^2\alpha+\cos^2\alpha=1\Rightarrow$

$10\cos^2\alpha-12\cos\alpha+3=0$ (II)

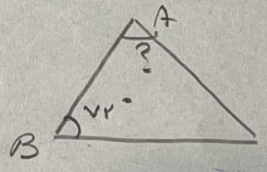
(I): $10\cos^2\alpha-12\cos\alpha-5=5(10\cos^2\alpha-12\cos\alpha+3)-8=0-8=-8$

سؤال ⑩

$\sin B\sin C=\cos^2\frac{A}{2}$

$B=75$

$A=?$



$\hat A+\hat B+\hat C=180$

$C=180-A-75=105-A$

$\sin B\sin C=\sin 75\sin(105-A)=$

$\sin 75^\circ(\sin 105\cos A-\cos 105\sin A)\Rightarrow\cos 105=-\cos 75$

$\sin 105=\sin 75$

$\Rightarrow\frac{1}{2}\left[\cos(A-2A)-\frac{\cos(105-A)}{-\cos A}\right]=\cos^2\frac{A}{2}$

$\frac{1}{2}(\cos(A-2A)+\cos A)=\cos^2\frac{A}{2}=\frac{1+\cos A}{2}$

$\cos(A-2A)+\cos A=1+\cos A\Rightarrow\cos(A-2A)=1$

$A-2A=0$ $\quad A=2A$

سوال ۸

$1-2\sin^2 15^\circ = \cos 30^\circ$

$2\cos^2 15^\circ - 1 = \cos 30^\circ$

فرمول‌های

$\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ} = \frac{\cos^2 20^\circ - \sin^2 20^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} = \cos 40^\circ$

$\cos(2\times 20^\circ) = \cos 40^\circ$

$\underbrace{(8\sin 10^\circ)\cos 10^\circ}_{\sin 20^\circ} \times \cos 20^\circ \times \cos 40^\circ =$

$\frac{1}{2}\sin 20^\circ \times \cos 20^\circ \times \cos 40^\circ$

$\frac{1}{2}\times\frac{1}{2}\sin 40^\circ \times \cos 40^\circ =$

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\sin 80^\circ = \frac{\frac{1}{8}\sin 80^\circ}{\sin 10^\circ} = \frac{1}{8}\cot 10^\circ$

$\sin 80^\circ = \cos 10^\circ$

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$\sqrt{1+\sin 4^\circ} = \sqrt{1+\cos 86^\circ}$

فرمول‌های

$1+\cos 86^\circ = 2\cos^2 43^\circ$

$\sqrt{2\cos^2 43^\circ} = \sqrt{2}\,|\cos 43^\circ| = \sqrt{2}\cos 43^\circ$

$\sin 47^\circ + \sin 13^\circ = \sin(30^\circ+17^\circ) + \sin(30^\circ-17^\circ) =$

$\sin 30^\circ\cos 17^\circ + \sin 17^\circ\cos 30^\circ + \sin 30^\circ\cos 17^\circ - \sin 17^\circ\cos 30^\circ = 2(\frac{1}{2})\cos 17^\circ = \cos 17^\circ$

$\frac{\sqrt{2}\cos 43^\circ}{\cos 43^\circ} = \boxed{\sqrt{2}}$