آفرین ۲۰

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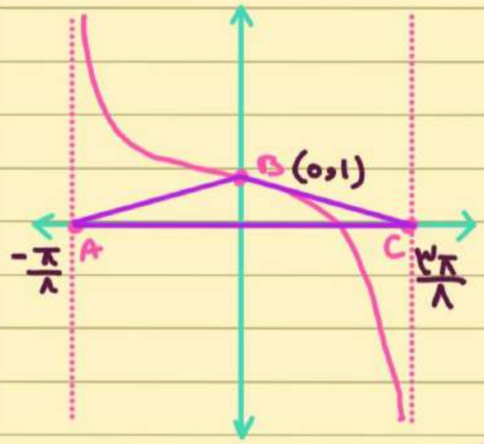
ب

$$y=\tan\left(-2x+\frac{\pi}{4}\right)$$



$$\xrightarrow{x=0} \tan\left(-2(0)+\frac{\pi}{4}\right)=1$$

$$B=(0,1)$$

$\tan\left(-2x+\frac{\pi}{4}\right) =$ تعریف نشده

$$-2x+\frac{\pi}{4}=\frac{\pi}{2} \longrightarrow x=-\frac{\pi}{8} \longrightarrow C$$

$$-2x+\frac{\pi}{4}=-\frac{\pi}{2} \longrightarrow x=\frac{3\pi}{8} \longrightarrow A$$

$$S_{\overset{\triangle}{ABC}}=\frac{\cancel{\left(\frac{3\pi}{8}-\left(-\frac{\pi}{8}\right)\right)}^{\frac{\pi}{2}}\times 1}{2}=\frac{\pi}{4}$$

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$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0$

$\xrightarrow{x=\frac{2}{3}} \left(\frac{2}{3}\right)^2-\frac{2}{3}(\sin\alpha)-\frac{1}{4}\cos^2\alpha=0$ ۲

$\xrightarrow{\times 36} 16-24\sin\alpha-9\underbrace{\cancel{\cos^2\alpha}}_{(1-\sin^2\alpha)}=0$

$9\sin^2\alpha-24\sin\alpha+7=0 \xrightarrow{\Delta=b^2-4ac} (-24)^2-4(9\times 7)=324=18^2$

$\sin\alpha \to \frac{24+18}{18}$ غ ق $(\sin\alpha\leq 1)$

$\sin\alpha \to \frac{24-18}{18}=\frac{1}{3}$ ✓

$\sin^2\alpha+\cos^2\alpha=1 \longrightarrow \frac{1}{9}+\cos^2\alpha=1 \longrightarrow \cos^2\alpha=\frac{8}{9}$

$\tan^2\alpha+1=\frac{1}{\cos^2\alpha}=\frac{9}{8}$ ✓

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$\log(\sin\alpha)-\log(-\cos\alpha)=\log\left(\frac{-\sin\alpha}{\cos\alpha}\right)=\log(-\tan\alpha)=\log 3$ ۲

$\tan\alpha=-3 \quad 1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \quad 1+9=\frac{1}{\cos^2\alpha} \longrightarrow \cos\alpha=\frac{-\sqrt{10}}{10}$

(کسینوس منفی است تا رادیکال تعریف شود.)

$\sin^2\alpha+\cos^2\alpha=1 \longrightarrow \sin^2\alpha=\frac{9}{10} \longrightarrow \sin\alpha=\frac{3\sqrt{10}}{10}$

$\sin\alpha-\cos\alpha=\frac{3\sqrt{10}}{10}+\frac{\sqrt{10}}{10}=\frac{4\sqrt{10}}{10}=\frac{2\sqrt{10}}{5}$ ✓

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$$(1-\sin^2\theta)$$

$$\sin^4\alpha + \cancel{\cos^4\theta} = \frac{4}{5} \to \sin^4\theta - \sin^2\theta + 1 = \frac{4}{5} \ *$$

$$\cancel{\cos^4\theta} + \sin^2\theta = \sin^4\theta - 2\sin\theta + 1 + \sin^2\theta$$

$$(1-\sin^2\theta)^2$$

$$\underbrace{\sin^4\theta - \sin\theta + 1}_{*} = \frac{4}{5}$$

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$$\frac{3\sin x + \cos x \sin x}{\cancel{2-2\cos^2 x}} = \frac{\cancel{\sin x}(3+\cos x)}{\sin^2 x} \Rightarrow \frac{\overbrace{\cancel{3+\cos x}}^{\text{همواره مثبت}}}{\sin x} < 0$$

$$2(1-\cos^2 x)$$

$$\sin x < 0$$

$$\sin x \tan x - \frac{1}{\cos x} > 0 \longrightarrow \frac{\sin^2 x - 1}{\cos x} > 0 \to \frac{-\cos^2 x}{\cos x} > 0 \to \cos x < 0$$

$\sin x, \cos x < 0 \longrightarrow$ انتهای کمان x در ناحیه‌ی سوم مثلثاتی

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$$\frac{2}{\sin x} + \frac{3}{\cos x} = \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0$$

$$2\cos x + 3\sin x = 0 \Longrightarrow \frac{\sin x}{\cos x} = -\frac{2}{3}$$

$$\tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = -\frac{13}{6}$$

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$$1+\sin\theta = \left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2 \longrightarrow \sqrt{\sin 50^\circ + 1} = \sin 25^\circ + \cos 25^\circ$$

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$$\sin x+\cos x = \sqrt{2}\,\sin\left(x+\frac{\pi}{4}\right)$$

$$\sin 25^\circ+\cos 25^\circ = \sqrt{2}\,\sin 70^\circ = \sqrt{2}\cos 20^\circ$$

$$\sin p+\sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\sin 50^\circ+\sin 10^\circ = 2\,\overset{\frac{1}{2}}{\cancel{\sin 30^\circ}}\cos 20^\circ = \cos 20^\circ$$

$$\Longrightarrow \frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ} = \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \sqrt{2}$$

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$$\left(\overset{\cos 10^\circ}{\cancel{1-2\sin^2 5^\circ}}\right)\left(\overset{\cos 20^\circ}{\cancel{2\cos^2 10^\circ-1}}\right)\left(\overset{\cos 40^\circ}{\cancel{\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}}}\right) = \cos 10^\circ\cos 20^\circ\cos 40^\circ$$

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$$\xrightarrow{\times\frac{\sin 10^\circ}{\sin 10^\circ}} \frac{\overset{\frac{1}{2}\sin 20^\circ\ \ \frac{1}{4}\sin 40^\circ\ \ \frac{1}{8}\sin 80^\circ}{\cancel{\sin 10^\circ\cos 10^\circ\cos 20^\circ\cos 40^\circ}}}{\sin 10^\circ} = \frac{\overset{\cos 10^\circ}{\cancel{\sin 80^\circ}}}{8\sin 10^\circ} = \frac{1}{8}\cot 10^\circ$$

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$$\sin x + 3\cos x = 2 \longrightarrow \sin x = 2 - 3\cos x$$

$$\rightarrow (\sin x + 3\cos x)^2 = \sin^2 x + 6\sin x\cos x + 9\cos^2 x = 4$$

$$(1-\cos^2 x) + 9\cos^2 x + 6(2-3\cos x)\cos x = 4$$

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$$-10\cos^2 x + 12\cos x = 3 \; *$$

$$5\cos 2x - 12\cos x = 5(\cos^2 x - 1) - 12\cos x = \underset{*}{\overset{-3}{\cancel{10\cos^2 x - 12\cos x}}} - 5 = -8$$

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$$\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2} = \frac{1+\cos\hat{A}}{2}$$

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$$2\sin\hat{B}\cdot\sin\hat{C} = 1+\cos\hat{A}$$

$$2\left(\frac{1}{2}\left(\cos(\hat{B}-\hat{C}) - \cos(\pi-\hat{A})\right)\right) = \cos\hat{A}+1$$

$$\cos(\hat{B}-\hat{C}) + \cos\hat{A} = \cos\hat{A}+1$$

$$\cos(\underset{0}{\cancel{\hat{B}-\hat{C}}}) = 1$$

$$\hat{B} = \hat{C} = 72^\circ \qquad 180^\circ - 144 = 36^\circ = \hat{A}$$