لیمیا امیری‌سا

۱۹٫۵ آفرین

(1) ۱٫۵

$y=\tan(-2x+\frac{\pi}{4})$ $S_{ABC}=\frac{\pi}{2}\times 1\times \frac{1}{2}=\frac{\pi}{4}$

$\lim_{x\to A}\tan(-2x+\frac{\pi}{4})=+\infty$

$x=0 \to y=\tan\frac{\pi}{4}=1 \to \boxed{B=1}$

$-2x+\frac{\pi}{4}=\frac{\pi}{2}\to \boxed{A=-\frac{\pi}{8}}$

$\tan(-2x+\frac{\pi}{4})=0 \to x=\frac{\pi}{8} \to \boxed{C=\frac{\pi}{8}}$

$-2x+\frac{\pi}{4}=\pm\frac{\pi}{2}$ یا $x=-\frac{\pi}{8}$

یا $x=\frac{3\pi}{8}$

$S=\frac{AC\times B}{2}=\frac{\frac{\pi}{4}\times 1}{2}=\frac{\pi}{8}$

قاعده $=\frac{4\pi}{8}=\frac{\pi}{2}$

(2) ۲

$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}\underbrace{\cos^2\alpha}_{(1-\sin^2\alpha)}=0$

$\frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0 \xrightarrow{\times 36} 9\sin^2\alpha-24\sin\alpha+7=0 \to \sin\alpha=\frac{21}{9}$ غ ق ق

$\to \sin\alpha=\frac{3}{9}=\boxed{\frac{1}{3}}$ ✓

$\S\ 1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \to \cos^2\alpha=1-\sin^2\alpha=1-\frac{1}{9}=\frac{8}{9}$

$\to 1+\tan^2\alpha=\frac{9}{8}$ ✓

(3) ۲

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3 \to \log\frac{\sin}{-\cos}=\log 3$

$\frac{\sin}{-\cos}=3 \to \frac{\sin}{\cos}=\tan=-3 \to \S\ \cos^2=\frac{1}{1+\tan^2}=\frac{1}{10}\to \cos=\frac{-\sqrt{10}}{10}$

★کسینوس منفی است چون $-\cos_x$ باید در دامنهٔ لگاریتم باشد!

$\sin=\sqrt{1-\frac{1}{10}}=\frac{3\sqrt{10}}{10}$ $\sin\alpha-\cos\alpha=\frac{4\sqrt{10}}{10}$

(4) ۲

$\sin^4+\cos^4=\frac{4}{5} \xrightarrow{\cos^2\alpha=1-\sin^2\alpha} \sin^4\alpha-\sin^2\alpha+1=\frac{4}{5}$

طبق بالایی

$\cos^4\alpha+\sin^2\alpha= \xrightarrow{\cos^4\alpha=(1-\sin^2\alpha)^2} 1+\sin^4\alpha-2\sin^2\alpha+\sin^2\alpha=\sin^4\alpha-\sin^2\alpha+1=\frac{4}{5}$

$\S$

(5) ۲

$\frac{2\sin x+\sin x\cos x}{\underset{2(1-\cos^2x)}{2-2\cos^2 x}}=\frac{\cancel{\sin x}\overbrace{(2+\cos x)}^{\text{همواره مثبت}}}{2\sin^{\cancel{2}}x}<0 \to 2\sin x<0 \to \boxed{\sin x<0}$

ناحیه سوم

$\sin x\tan x-\frac{1}{\cos x}=\frac{\sin^2 x-1}{\cos x}=\frac{-\cos^2 x}{\cos x}=-\cos x>0 \to \boxed{\cos x<0}$ ✓

(6)

$$\frac{2}{\sin x}+\frac{3}{\cos x}=\frac{2\cos x+3\sin x}{\sin x\cos x}=0 \rightarrow 2\cos x+3\sin x=0 \xrightarrow{\div\cos x} 2+3\tan x=0$$

$$\tan x=\frac{-2}{3} \qquad \cot x=\frac{1}{\tan x}=\frac{-3}{2} \qquad \tan x+\cot x=\frac{-2}{3}-\frac{3}{2}=\frac{-4-9}{6}=\frac{-13}{6}$$

(7)

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ} \xrightarrow[\sin 50^\circ=\cos 40^\circ]{50=30+20 \quad 10=30-20} \frac{\sqrt{1+\cos 40}}{\sin 30\cos 20+\cancel{\cos 30\sin 20}+\sin 30\cos 20-\cancel{\sin 20\cos 30}}$$

$$=\frac{\sqrt{2\cos^2 20}}{2\sin 30\cos 20}=\frac{\sqrt{2}\cancel{\cos 20}}{2\sin 30\cancel{\cos 20}}=\frac{\sqrt{2}}{2}\times 2=\sqrt{2}$$

$$1-2\sin^2 5=\cos 10 \qquad \overset{\text{فرمول طلایی}}{2\cos^2 10-1=\cos 20} \qquad \frac{1-\tan^2 10}{1+\tan^2 10}=\cos 40$$ (8)

$$\text{عبارت}=\cos 10\cos 20\cos 40 \xrightarrow[\div\sin 10]{\times\sin 10}=\frac{\sin 10\cos 10\cdot\cos 20\cdot\cos 40}{\sin 10}$$

ارفاق !! $\cot 10$

$$=\frac{\frac{1}{2}\cos 20\sin 20\cdot\cos 40}{\sin 10}=\frac{\frac{1}{4}\cos 40\sin 40}{\sin\cos 10}=\frac{\frac{1}{8}\sin 80}{\sin 10}=\frac{\frac{1}{8}\cos 10}{\sin 10}=\frac{1}{8}\cancel{\tan 10}$$

(9)

$$\cancel{\sin\alpha+2\sqrt{3}\cos\alpha=\ldots \rightarrow \ldots\sin^2\alpha+\sin\alpha=2 \rightarrow \ldots\sin^2\alpha-\sin\alpha-1=0}$$

$$\sin\alpha+3\cos\alpha=2 \rightarrow \sin\alpha=2-3\cos\alpha \xrightarrow{\text{توان ۲}} \sin^2\alpha=4+9\cos^2\alpha-12\cos\alpha$$

$$\xrightarrow{+\cos^2\ \text{دو طرف}} 1=10\cos^2\alpha-12\cos\alpha+4 \rightarrow \underbrace{10\cos^2\alpha}_{5(1+\cos 2\alpha)}-12\cos\alpha+3=0$$

$$\rightarrow 5+\underbrace{5\cos 2\alpha-12\cos\alpha}_{=-8}+\cancel{3}=0$$

(10)

$$\sin B\sin C=\cos^2\frac{A}{2} \xrightarrow[\text{مکمل}]{A+B \text{ و } C} \sin B\sin(A+B)=\cos^2\frac{A}{2} \xrightarrow{\text{طلایی}}$$

$$\sin B(\sin A\cos B+\cos A\sin B)=\frac{1}{2}+\frac{\cos A}{2} \rightarrow \sin A\sin B\cos B+\cos A\sin^2 B-\frac{\cos A}{2}=\frac{1}{2}$$

$$\frac{1}{2}\sin 2B\sin A+\cos A\underbrace{\left(\sin^2 B-\frac{1}{2}\right)}_{\frac{-\cos 2B}{2}}=\frac{1}{2} \rightarrow \sin 2B\sin A-\cos 2B\cos A=1$$

$$\rightarrow -\cos(A+2B)=1 \rightarrow A+2B=180^\circ \rightarrow A+144=180 \rightarrow A=36$$