نام: دنیا احمدی — شماره تکلیف: ۱۵ — مقطع: دوازدهم دختران B — آزمون ۲۰

①

$T=\frac{\pi}{|-2|}=\frac{\pi}{2}=AC$ طول $\qquad B\,|\,y=\tan(\frac{\pi}{4})=1 \to S_{ABC}=\frac{1\times\frac{\pi}{2}}{2}=\boxed{\frac{\pi}{4}}$

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②

$A\,|\,\frac{2}{3} \to \frac{4}{9}-(\sin\alpha)\frac{2}{3}-\frac{1}{4}\cos^2\alpha=0 \to \frac{4}{9}-\sin\alpha(\frac{2}{3})-\frac{1}{4}(1-\sin^2\alpha) \to \sin\alpha=\frac{7}{3}\times$

$\sin\alpha=\frac{1}{3}\checkmark$

$\to \cos^2\alpha=1-\sin^2\alpha=1-\frac{1}{9}=\frac{8}{9} \to 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\boxed{\frac{9}{8}}$

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③

$\log^{\sin\alpha}-\log^{-\cos\alpha}=\log^{3} \to \log(\frac{-\sin\alpha}{\cos\alpha})=\log^{3} \to \frac{-\sin\alpha}{\cos\alpha}=3 \to \tan\alpha=-3$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \to \cos^2\alpha=\frac{1}{10} \to \cos\alpha=\frac{-\sqrt{10}}{10},\ \sin=\frac{3}{\sqrt{10}} \to \sin\alpha-\cos\alpha=\frac{4}{\sqrt{10}}=\boxed{\frac{2\sqrt{10}}{5}}$

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④

$\sin^4\alpha+(1-\sin^2\alpha)=\frac{4}{5} \to \sin^4\alpha-\sin^2\alpha+1=\frac{4}{5}$

$\cos^4\alpha+\sin^2\alpha=(1-\sin^2\alpha)^2+\sin^2\alpha=\sin^4\alpha-\sin^2\alpha+1=\boxed{\frac{4}{5}}$

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⑤

$\frac{\sin(3+\cos\alpha)}{2\underbrace{(1-\cos^2\alpha)}_{\sin^2\alpha}}<0 \to \frac{\overbrace{3+\cos\alpha}^{\text{همواره مثبت}}}{2\sin\alpha}<0 \to \sin\alpha<0$

$\frac{\sin^2\alpha-1}{\cos\alpha}=\frac{-(1-\sin^2\alpha)}{\cos\alpha}=\frac{-\cos^2\alpha}{\cos\alpha}=-\cos\alpha>0 \to \cos\alpha<0$

→ ناحیه سوم

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⑥

$\frac{2}{\sin\alpha}=\frac{-3}{\cos\alpha} \to \frac{\sin\alpha}{\cos\alpha}=\frac{2}{-3} \to \tan\alpha+\cot\alpha=\frac{-2}{3}+(-\frac{3}{2})=\boxed{\frac{-13}{6}}$

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⑦

$\frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}=\frac{\sqrt{\sin 90+\sin 50}}{\sin 50+\sin 10}$ تبدیل به ضرب

$\begin{cases} 2\sin(\frac{90+50}{2})\cos(\frac{90-50}{2})=2\sin 70\cos 20=2\cos^2 20 \\ 2\sin(\frac{50+10}{2})\cos(\frac{50-10}{2})=2\times\frac{1}{2}\times\cos 20=\cos 20 \end{cases} \to \frac{\sqrt{2\cos^2 20}}{\cos 20}=\boxed{\sqrt{2}}$

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⑧

$\cos 2(5)\times\cos 2(10)\times\cos 2(20)=\cos 10\cdot\cos 20\cdot\cos 40\times\frac{\sin 10}{\sin 10}=\frac{1}{2}\frac{\sin 20\cos 20\cos 40}{\sin 10}=\frac{1}{4}\frac{\sin 40\cos 40}{\sin 10}$

$=\frac{\frac{1}{8}\sin 80}{\sin 10}=\boxed{\frac{1}{8}\cot 10}$

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⑨

$\cos\alpha=t \longrightarrow \sin\alpha=2-3t \to \sin^2\alpha+\cos^2\alpha=1 \to (2-3t)^2+t^2=1 \to 10t^2-12t+3=0 \to 10t^2-12t=-3 \ *$

$y=5\cos 2\alpha-12\cos\alpha=5(2t^2-1)-12t=10t^2-12t-5 \overset{*}{=} -3-5=\boxed{-8}$

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⑩

$\cos\frac{\hat{A}}{2}=\frac{1+\cos A}{2}$ ① $\qquad \sin B\sin C=\frac{\cos(B-C)+\overbrace{\cos(B+C)}^{\cos A}}{2}$ ② $\to$ ① = ② $\to 1=\cos(B-C)$

$B-C=0 \to B=C=72 \to \hat{A}=180-2(72)=\boxed{36^\circ}$

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