تینا جعفرپور — تکلیف ۱۵ — دوازدهم دختر B

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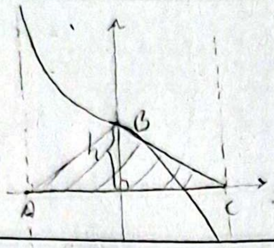

$y=\tan(-2x+\frac{\pi}{4})$ $\qquad \tan(-2(0)+\frac{\pi}{4})=\tan\frac{\pi}{4}=1=B$

$T=\frac{\pi}{|-2|}=\frac{\pi}{2} \Rightarrow |AC|=\frac{\pi}{2}$ $\qquad S_{ABC}=\frac{|AC|\times B}{2}=\frac{\frac{\pi}{2}\times 1}{2}=\boxed{\frac{\pi}{4}}$

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$x^2-(\sin\alpha)x-\frac{1}{4}\cos^2\alpha=0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}\cos^2\alpha=0 \xrightarrow{\cos^2\alpha=1-\sin^2\alpha} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36}=0$

$\xrightarrow{\times 36} 9\sin^2\alpha-24\sin\alpha+7=0=(3\sin\alpha-1)(3\sin\alpha-7) \Rightarrow \begin{cases}\sin\alpha=\frac{7}{3} \text{ غ ق ق} \\ \sin\alpha=\frac{1}{3} \Rightarrow \cos^2\alpha=1-\sin^2\alpha=\frac{8}{9}\end{cases}$ $\Rightarrow \boxed{1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{9}{8}}$

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$\log(\sin\alpha)-\log(-\cos\alpha)=\log\frac{\sin\alpha}{-\cos\alpha}=\log^{-\tan\alpha}=\log^3 \Rightarrow -\tan\alpha=3 \Rightarrow \tan\alpha=-3$

می‌دانیم $x>0$, $\log x$ در نتیجه $\left.\begin{array}{l}\sin\alpha>0 \\ -\cos\alpha>0 \Rightarrow \cos\alpha<0\end{array}\right\}$

$\Rightarrow \sin2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{2(-3)}{1+(-3)^2}=\frac{-6}{10}$

با توجه به علامت ها حاصل عبارت مثبت است

$(\sin\alpha-\cos\alpha)^2=\sin^2\alpha+\cos^2\alpha-2\sin\alpha.\cos\alpha=1-\sin2\alpha=1-(\frac{-6}{10})=\frac{16}{10} \Rightarrow \sin\alpha-\cos\alpha=\pm\sqrt{\frac{16}{10}}$

$\Rightarrow \sin\alpha-\cos\alpha=\sqrt{\frac{16}{10}}=\frac{4}{\sqrt{10}}=\frac{4\sqrt{10}}{10}=\boxed{\frac{2\sqrt{10}}{5}}$

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$\sin^4\theta+\underbrace{\cos^2\theta}_{1-\sin^2\theta}=\frac{6}{5}=\sin^4\theta-\sin^2\theta+1 \Rightarrow \sin^2\theta-\sin^4\theta=\frac{1}{5}$ $\qquad \underbrace{\cos^4\theta}_{1-\sin^4\theta}+\sin^2\theta=1-\sin^4\theta+\sin^2\theta=1+\frac{1}{5}=\boxed{\frac{6}{5}}$

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$\frac{3\sin x+\sin x\cos x}{2-2\cos^2x}<0 \Rightarrow \frac{\sin x(3+\cos x)}{2(\underbrace{1-\cos^2x}_{\sin^2x})}<0 \underset{\sin x\neq 0}{\Rightarrow} \frac{3+\cos x}{2\sin x}<0$ $\qquad -1\leq\cos x\leq 1 \Rightarrow 2\leq 3+\cos x\leq 4 \Rightarrow \sin x<0$

صورت همواره مثبت

$\sin x\tan x-\frac{1}{\cos x}>0 \Rightarrow \frac{1}{\cos x}(\underbrace{\sin^2x-1}_{-\cos^2x})>0 \xrightarrow{\cos x\neq 0} -\cos x>0 \Rightarrow \cos x<0$

$\Rightarrow$ ناحیه سوم

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$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \xrightarrow{\times\cos x} 2\cot x+3=0 \Rightarrow \cot x=\frac{-3}{2} \Rightarrow \tan x=\frac{-2}{3} \Rightarrow \tan x+\cot x=\frac{-2}{3}+\frac{-3}{2}=\boxed{\frac{-13}{6}}$

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$\frac{\sqrt{1+\sin50^\circ}}{\sin50^\circ+\sin10^\circ}=\frac{\sqrt{1+\cos40^\circ}}{\sin(30^\circ+20^\circ)+\sin(30^\circ-20^\circ)}=\frac{\sqrt{2\cos^2 20^\circ}}{\frac{1}{2}\cos20^\circ+\cancel{\frac{\sqrt{3}}{2}\sin20^\circ}+\frac{1}{2}\cos20^\circ-\cancel{\frac{\sqrt{3}}{2}\sin20^\circ}}=\frac{\sqrt{2}\cos20^\circ}{\cos20^\circ}=\boxed{\sqrt{2}}$

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$(1-2\sin^2 5^\circ)(2\cos^2 10^\circ-1)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)=(\cos10^\circ)(\cos20^\circ)(\cos40^\circ)=A \xrightarrow{\times\sin10^\circ} \underbrace{(\sin10^\circ)(\cos10^\circ)}_{\frac{1}{2}\sin20^\circ}(\cos20^\circ)(\cos40^\circ)=A.\sin10^\circ$

$\Rightarrow \frac{1}{2}\underbrace{(\sin20^\circ)(\cos20^\circ)}_{\frac{1}{2}\sin40^\circ}(\cos40^\circ)=\frac{1}{4}(\sin40^\circ)(\cos40^\circ)=\frac{1}{8}\underbrace{\sin80^\circ}_{\sin(\frac{\pi}{2}-10^\circ)}=A\sin10^\circ \Rightarrow \frac{1}{8}\cos10^\circ=A\sin10^\circ \xrightarrow{\div\sin10^\circ} \boxed{\frac{1}{8}\cot10^\circ=A}$

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$\sin\alpha+3\cos\alpha=2 \Rightarrow \sin\alpha=2-3\cos\alpha \Longrightarrow$ جایگذاری $\sin^2\alpha+\cos^2\alpha=1=(2-3\cos\alpha)^2+\cos^2\alpha=10\cos^2\alpha-12\cos\alpha+4=1$

$\Rightarrow 10\cos^2\alpha-12\cos\alpha=-3$

$5\cos2\alpha-12\cos\alpha=5(2\cos^2\alpha-1)-12\cos\alpha=10\cos^2\alpha-12\cos\alpha-5=(-3)-5=\boxed{-8}$

$\sin\hat{B}\sin\hat{C}=\cos^2\frac{\hat{A}}{2}=\frac{1+\cos\hat{A}}{2} \Rightarrow 2\sin\hat{B}\sin\hat{C}-\cos\hat{A}=1=2\sin\hat{B}.\sin\hat{C}-\underbrace{\cos(180-(\hat{B}+\hat{C}))}_{-\cos(\hat{B}+\hat{C})}=2\sin\hat{B}.\sin\hat{C}+\cos(\hat{B}+\hat{C})=1$

$\Rightarrow 2\sin\hat{B}.\sin\hat{C}+\cos\hat{B}.\cos\hat{C}-\sin\hat{B}\sin\hat{C}=\sin\hat{B}.\sin\hat{C}+\cos\hat{B}.\cos\hat{C}=\cos(\hat{B}-\hat{C})=1$

$\Rightarrow \cos(72^\circ-\hat{C})=1 \Rightarrow 72^\circ-\hat{C}=0 \Rightarrow C=72^\circ \Rightarrow A=180^\circ-(\hat{B}+\hat{C})=180^\circ-(72^\circ+72^\circ)=\boxed{36^\circ}$

از آنجایی که زاویه مثلث است