آرمانی کیا - تکلیف ۱۵

1. $x=0 \to \tan(\frac{\pi}{4})=1 \Rightarrow B|_1$ $\quad T=\frac{\pi}{|-2|}=\frac{\pi}{2} \Rightarrow AC=\frac{\pi}{2}$ $\quad S_{ABC}=\frac{AC\times B}{2}=\frac{\frac{\pi}{2}\times 1}{2}=\frac{\pi}{4}$

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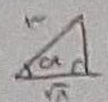
2. $x^2-(\sin a)x-\frac{1}{4}\cos^2 a=0$ $\quad (\frac{2}{3},0) \to \frac{4}{9}-\frac{2}{3}\sin a-\frac{1}{4}\underbrace{\cos^2 a}_{1-\sin^2 a}=0 \Rightarrow \frac{1}{4}\sin^2 a-\frac{2}{3}\sin a+\frac{7}{36}=0 \xrightarrow{\times 36}$

$9\sin^2 a-24\sin a+7=0 \Rightarrow \sin^2 a-24\sin a+63=0 \to (\sin a-21)(\sin a-3)=0$ $\to \sin a=\frac{1}{3}$ ✓ $\quad \to \sin a=\frac{7}{3}$ غ ق ق

$1+\tan^2 a=\frac{1}{\cos^2 a} \Rightarrow \frac{1}{(\frac{\sqrt{8}}{3})^2}=\frac{9}{8}$



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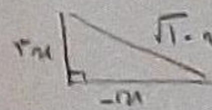
3. $\log\sin a-\log(-\cos a)=\log\frac{\sin a}{-\cos a}=\log 3 \Rightarrow -\tan a=3$



$\sin x-\cos x=\frac{4}{\sqrt{10}}$ $\quad (\sin x \to \frac{3}{\sqrt{10}},\ \cos x \to -\frac{1}{\sqrt{10}})$

* توجه داریم که طبق شرط دامنه ($\underbrace{-\cos a>0}_{\cos a<0}$, $\sin a>0$) : sin را مثبت، cos را منفی در نظر بگیریم

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4. $\cos^2\theta=1-\sin^2\theta \Rightarrow \sin^4\theta-\sin^2\theta+1=\frac{4}{5} \xrightarrow{\sin^2\theta=t} t^2-t+\frac{1}{5}=0 \xrightarrow{\Delta} t=\frac{1\pm\sqrt{1-\frac{4}{5}}}{2}=\frac{1\pm\sqrt{\frac{1}{5}}}{2}$

$t_1=\frac{1-\sqrt{\frac{1}{5}}}{2} \xrightarrow{\cos^2 a=1-\sin^2 a} \cos^2 a=\frac{1-\sqrt{\frac{1}{5}}}{2} \Rightarrow \cos^4 a=\frac{1+\frac{1}{5}-\frac{2}{\sqrt{5}}}{4}=\frac{6-2\sqrt{5}}{20} \xrightarrow{+\sin^4\theta} \frac{6-2\sqrt{5}}{20}+\frac{10+2\sqrt{5}}{20}=\frac{4}{5}$

$t_2=\frac{1-\sqrt{\frac{1}{5}}}{2} \xrightarrow{\cos^2 a=1-\sin^2 a} \cos^2 a=\frac{1+\sqrt{\frac{1}{5}}}{2} \Rightarrow \cos^4 a=\frac{1+\frac{1}{5}+\frac{2}{\sqrt{5}}}{4}=\frac{6+2\sqrt{5}}{20} \xrightarrow{+\sin^4\theta} \frac{6+2\sqrt{5}}{20}+\frac{10-2\sqrt{5}}{20}=\frac{4}{5}$

در هر صورت جواب نهایی $\frac{4}{5}$ خواهد بود.

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5. $\frac{\sin x(3+\cos x)}{2(\underbrace{1-\cos^2 x}_{\sin^2 x})}<0 \xrightarrow[-1\le\cos x\le 1]{\sin^2 x>0} \sin x<0$ $\quad (I)$

$\sin x\times\frac{\sin x}{\cos x}-\frac{1}{\cos x}>0 \Rightarrow \frac{\sin^2 x-1}{\cos x}>0 \to -\cos x>0$ یا $\cos x<0$ $\quad (II)$

$\xrightarrow{I, II}$ ناحیه سوم

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6. $\frac{2\cos x+3\sin x}{\sin x\cos x}=0 \xrightarrow{\sin x,\cos x\neq 0} 2\cos x+3\sin x=0 \Rightarrow \cos x=-\frac{3\sin x}{2} \rightsquigarrow \cos^2 x+\sin^2 x=1$

$\frac{9}{4}\sin^2 x+\sin^2 x=1 \Rightarrow \frac{13}{4}\sin^2 x=1 \Rightarrow \sin^2 x=\frac{4}{13} \xrightarrow[\sin x>0]{\text{فرض می‌کنیم}} \sin x=\frac{2}{\sqrt{13}} \Rightarrow \cos x=\frac{-3\times 2}{2\times\sqrt{13}}=\frac{-3}{\sqrt{13}}$

$\tan x+\cot x=\frac{1}{\sin x\cos x}=\frac{1}{\frac{2}{\sqrt{13}}\times\frac{-3}{\sqrt{13}}} \Rightarrow \frac{13}{-6}$

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7. صورت: $\sin 50=\cos 40 \Rightarrow 1+\cos 40=2\cos^2 20 \Rightarrow \sqrt{2\cos^2 20}=\sqrt{2}\cos 20$

$\sin 50+\sin 10=2\underbrace{\sin 30}_{\frac{1}{2}}\cos 20 \Rightarrow \sin 50+\sin 10=\cos 20$

کسر $=\frac{\sqrt{2}\cos 20}{\cos 20}=\sqrt{2}$

8. $1-2\sin^2\alpha = \cos 2\alpha \Rightarrow 1-2\sin^2 5 = \cos 10$ $\quad 2\cos^2\alpha - 1 = \cos 2\alpha \Rightarrow 2\cos^2 10 - 1 = \cos 20$ $\quad \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha} = \cos 2\alpha$

$$\Rightarrow \frac{1-\tan^2 20}{1+\tan^2 20} = \cos 40 \qquad \Rightarrow \underbrace{\underbrace{\underbrace{(\times \sin 10)\cos 10}_{\frac{1}{2}\sin 20} \times \cos 20}_{\frac{1}{4}\sin 40} \times \cos 40}$$

$$\frac{1}{8}\sin 80 = \frac{1}{8}\cos 10 \xrightarrow{\div \sin 10} \frac{1}{8}\cot 10$$

۹. $\left.\begin{array}{l} \sin a = y \\ \cos a = x \end{array}\right\} \quad y + ۳x = ۲ \Rightarrow y = ۲ - ۳x$

$\sin^۲ a + \cos^۲ a = ۱ \Rightarrow (۲ - ۳x)^۲ + x^۲ = ۱$

$۴ + ۹x^۲ - ۱۲x + x^۲ = ۱ \Rightarrow ۱۰x^۲ - ۱۲x + ۳ = ۰$ *

$۵\cos ۲a - ۱۲\cos a = ۵(۲\cos^۲ a - ۱) - ۱۲\cos a = ۵(۲x^۲ - ۱) - ۱۲x = \underbrace{۱۰x^۲ - ۱۲x}_{-۳} - ۵ = -۸$

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