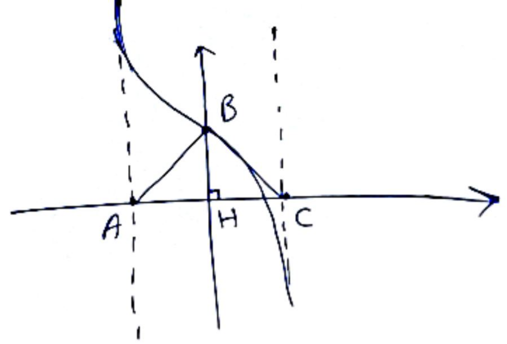
مثلثات — تکلیف شماره ۱۵



۱ـ

$$AC = \frac{\text{دوره}}{\text{ضریب}} = \frac{\pi}{|a|} = \frac{\pi}{2}$$

$$BH \longrightarrow x=0 \longrightarrow y = \tan\left(-2(0)+\frac{\pi}{4}\right)$$

$$y = \tan\left(\frac{\pi}{4}\right) = 1 \longrightarrow BH = 1$$

$$S_{\overset{\Delta}{ABC}} = \frac{\frac{\pi}{2}\times 1}{2} = \frac{\pi}{4}$$ جواب

۲ـ

$$x^2 - (\sin a)x - \frac{1}{4}\cos^2 a = 0$$

$$x = \frac{2}{3} \rightarrow y = 0 \longrightarrow \frac{4}{9} - \frac{2}{3}\sin a - \frac{1}{4}\cos^2 a = 0 \longrightarrow \frac{4}{9} - \frac{2}{3}\sin a - \frac{1}{4}(1-\sin^2 a) = 0$$

$$\longrightarrow \frac{4}{9} - \frac{2}{3}\sin a - \frac{1}{4} + \frac{1}{4}\sin^2 a = 0$$

$$\frac{1}{4}\sin^2 a - \frac{2}{3}\sin a + \frac{7}{36} = 0 \xrightarrow{\times 36} 9\sin^2 a - 24\sin a + 7 = 0$$

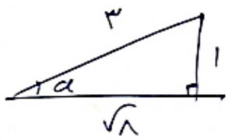
$$\Delta = 24^2 - 4(7)(9) = 324$$

$$\sqrt{\Delta} = 18$$

$$\sin a = \frac{24 \pm 18}{18}$$

$\rightarrow \sin a = 2{,}33$ — غ ق ق ✗

$\rightarrow \boxed{\sin a = \frac{1}{3}}$ ق ق



$$\cos a = \frac{\sqrt{8}}{3} \longrightarrow 1 + \tan^2 a = \frac{1}{\cos^2 a} = \frac{1}{\left(\frac{\sqrt{8}}{3}\right)^2} = \frac{9}{8}$$ جواب

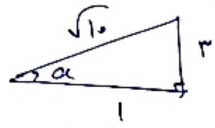
۳ـ

$$\log(\sin a) - \log(-\cos a) = \log 3 \xrightarrow[\text{دامنه}]{\text{با توجه به}} -\cos a > 0$$

$$\cos a < 0$$

$$\log\frac{\sin a}{-\cos a} = \log 3 \longrightarrow \frac{\sin a}{-\cos a} = 3 \longrightarrow \sin a = -3\cos a \xrightarrow{\div \cos a}$$

$$\tan a = -3$$



$$\cos a < 0 \longrightarrow \cos a = \frac{-1}{\sqrt{10}} \qquad \sin a = \frac{3}{\sqrt{10}}$$

$$\sin a - \cos a = \frac{3}{\sqrt{10}} - \left(\frac{-1}{\sqrt{10}}\right) = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$$ جواب

۴ـ

$$\sin^4\theta + \cos^2\theta = \frac{4}{5} \longrightarrow \sin^4\theta + (1-\sin^2\theta) = \frac{4}{5}$$

$$\sin^4\theta - \sin^2\theta + \frac{1}{5} = 0$$

$$\Delta = 1 - 4\left(\frac{1}{5}\right)(1) = \frac{1}{5}$$

$$\sin^2\theta = \frac{1 \pm \frac{\sqrt{5}}{5}}{2}$$

$\rightarrow \sin^2\theta = \dfrac{1+\frac{\sqrt{5}}{5}}{2}$ ✓

$\rightarrow \sin^2\theta = \dfrac{1-\frac{\sqrt{5}}{5}}{2}$ ✓

هر دو جواب برای $\sin^2\theta$ قابل قبول هستند و تفاوتی در پاسخ $\sin^4\theta+\cos^4\theta$ ایجاد نمی‌کنند.

نکته جالب

$$\text{if } \sin^2\theta = \frac{1+\frac{\sqrt{5}}{5}}{2} \longrightarrow \sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1+\frac{\sqrt{5}}{5}}{2} = \frac{1-\frac{\sqrt{5}}{5}}{2}$$

$$\cos^4\theta + \sin^2\theta = \left(\frac{1-\frac{\sqrt{5}}{5}}{2}\right)^2 + \frac{1+\frac{\sqrt{5}}{5}}{2} = \frac{1+\frac{1}{5}-\cancel{\frac{2\sqrt{5}}{5}}+2+\cancel{\frac{2\sqrt{5}}{5}}}{4} = \frac{3+\frac{1}{5}}{4} = \frac{4}{5}$$

جواب

۵-

$$\underbrace{\sin x \tan x}_{\frac{\sin^2 x}{\cos x}} - \frac{1}{\cos x} > 0 \longrightarrow \frac{\sin^2 x - 1}{\cos x} > 0 \longrightarrow \frac{-\cos^2 x}{\cos x} > 0$$

$$\cos x < 0$$

«صورت هم باید منفی باشد.»

$$\frac{3\sin x + \sin x\cos x}{\underbrace{2 - 2\cos^2 x}} < 0$$

$$0 < \cos^2 x \le 1 \rightarrow 0 > -2\cos^2 x \ge -2 \longrightarrow \underbrace{2 > 2-2\cos^2 x \ge 0}_{\text{مثبت}}$$

طبق صورت سوال

$$\cos x \neq 0$$

$$\longrightarrow 3\sin x + \sin x\cos x < 0 \longrightarrow \sin x(\underbrace{3+\cos x}_{\text{مثبت}}) < 0 \longrightarrow \sin x < 0$$

پس در ناحیه سوم قرار دارد.

۶-

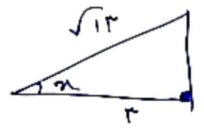
$$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \longrightarrow \frac{2\cos x + 3\sin x}{\sin x\cos x} = 0 \longrightarrow 2\cos x + 3\sin x = 0$$

$$\cos x = \frac{-3}{2}\sin x$$

cos و sin مختلف العلامت اند.

$$\sin^2 x + \cos^2 x = 1 \rightarrow \sin^2 x + \left(\frac{-3}{2}\sin x\right)^2 = 1$$

$$\frac{13}{4}\sin^2 x = 1 \rightarrow \sin^2 x = \frac{4}{13} \longrightarrow \sin x = \pm\frac{2}{\sqrt{13}}$$



$$\text{if } \sin x = \frac{2}{\sqrt{13}} \longrightarrow \cos x = \frac{-3}{\sqrt{13}}$$

$$\tan x + \cot x = \frac{2}{\sin 2x} = \frac{\cancel{2}}{\cancel{2}\sin x\cos x} = \frac{1}{\frac{2}{\sqrt{13}} \times \frac{-3}{\sqrt{13}}} = \frac{-13}{6}$$

جواب

[illegible]

-7

$$\frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10}$$

$$\sin 50 = \cos 40 = 2\cos^2 20 - 1 \longrightarrow \sqrt{1+\sin 50} = \sqrt{2\cos^2 20 \cancel{-1} \cancel{+1}} = \sqrt{2}\cos 20$$

$$\begin{cases} \sin 50 = \cos 40 = 2\cos^2 20 - 1 \\ \sin 10 = \cos 80 = 2\cos^2 40 - 1 = 2\left(2\cos^2 20 - 1\right)^2 - 1 = 8\cos^4 20 + 2 - \end{cases}$$

$$8\cos^2 20 - 1 = 8\cos^4 20 - 8\cos^2 20 + 1$$

$$\sin 50 + \sin 10 = 8\cos^4 20 - 8\cos^2 20 \cancel{+1} + 2\cos^2 20 \cancel{-1} = 8\cos^4 20 - 6\cos^2 20$$

$$\frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10} = \frac{\sqrt{2}\cos 20}{8\cos^4 20 - 6\cos^2 20} = \frac{\sqrt{2}\cancel{\cos 20}}{\cancel{\cos 20}\left(8\cos^3 20 - 6\cos 20\right)} = \frac{\sqrt{2}}{8\cos^3 20 - 6\cos 20}$$

-8

$$\left(1-2\sin^2 5\right)\left(2\cos^2 10 - 1\right)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)$$

$$\sin^2 5 = \frac{1+\cos 10}{2} \rightarrow 2\sin^2 5 = 1-\cos 10 \longrightarrow 1-2\sin^2 5 = \cos 10$$

$$2\cos^2 10 - 1 = \cancel{1} + \cos 20 \cancel{-1} = \cos 20$$

$$\frac{1-\tan^2 20}{1+\tan^2 20} = \frac{\dfrac{\cos^2 20 - \sin^2 20}{\cancel{\cos^2 20}}}{\dfrac{1}{\cancel{\cos^2 20}}} = \cos^2 20 - \sin^2 20 = \cos 40$$

$$= \cos 10 \cos 20 \cos 40 \xrightarrow[\text{بر } \sin 10]{\text{ضرب و تقسیم}} \underbrace{\underbrace{\underbrace{\sin 10 \cos 10}_{\frac{1}{2}\sin 20} \cos 20}_{\frac{1}{4}\sin 40} \cos 40}_{\frac{1}{8}\sin 80} = \frac{1}{8}\sin 80$$

$$= \frac{\frac{1}{8}\cos 10}{\sin 10} = \frac{1}{8}\cot 10$$

جواب

نگار بیات

۱۰-

$$۲\sin\hat{B}\sin\hat{C} = 1 + \cos A - (\cos(\pi - \hat{A}))$$

$$B + C$$

$$۲\sin\hat{B}\sin\hat{C} = 1 - \cos B\cos C + \sin B\sin C$$

$$1 = \sin\hat{B}\sin\hat{C} + \cos\hat{B}\cos\hat{C} \longrightarrow 1 = \cos(\hat{B} - \hat{C}) \longrightarrow \hat{B} - \hat{C} = ۰$$

$$\hat{B} = \hat{C} = ۷۲$$

$$\hat{A} = ۱۸۰ - (\hat{B} + \hat{C}) = ۱۸۰ - (۷۲ + ۷۲) = ۳۶$$

جواب