۱۹/۲۵ آفرین

نام و نام خانوادگی ......لیلی طباطبایی......... پاسخنامه تشریحی تکلیف شماره ...۱۵... کلاس دوازدهم – ...

۱

$A, C \to \tan(-2x+\frac{\pi}{4}) = 0 \Rightarrow -2x+\frac{\pi}{4} = k\pi \xrightarrow{If\ k=0} -2x+\frac{\pi}{4}=0 \Rightarrow x=\frac{\pi}{8} = C$

$If\ k=1 \Rightarrow -2x+\frac{\pi}{4}=\pi \Rightarrow x = -\frac{3\pi}{8} = A$ $\quad \begin{cases} C(\frac{\pi}{8}, 0) \\ A(-\frac{3\pi}{8}, 0) \end{cases} \quad B(0,1)$

$B|_{x=0} \Rightarrow y_B = \tan(-2\times 0 + \frac{\pi}{4}) = 1$

مساحت $\Delta = \frac{1}{2}|x_A(y_B-y_C)+x_B(y_C-y_A)+y_C(y_A-y_B)| = \frac{1}{2}|-\frac{3\pi}{8}+0-\frac{\pi}{8}| = \frac{1}{2}\times\frac{\pi}{2} = \boxed{\frac{\pi}{4}}$

(۲)

۲

$x^2-(\sin\alpha)x-\frac{1}{4}\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}=0 \Rightarrow \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{\sin^2\alpha}{4}=0 \Rightarrow 9\sin^2\alpha-24\sin\alpha+7=0$

$\sin\alpha = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{24\pm 18}{18}$ → $x_1=\frac{7}{3}$ بزرگتر از ۱ ✗ ، $x_2=\frac{1}{3}$ ✓ $=\sin\alpha$

$\cos^2\alpha = 1-\frac{1}{9}=\frac{8}{9}$

$\Rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = \boxed{\frac{9}{8}}$

(۲)

۳

$\log(\sin\alpha)-\log(-\cos\alpha)=\log 3 \Rightarrow \log(\frac{\sin\alpha}{-\cos\alpha})=\log 3 \Rightarrow \frac{\sin\alpha}{\cos\alpha}=-3=\tan\alpha$

$\sin\alpha-\cos\alpha=\cos\alpha(\frac{\sin\alpha}{\cos\alpha}-1)=\cos\alpha(\tan\alpha-1)=-4\cos\alpha$

$\sin^2\alpha+\cos^2\alpha=1 \Rightarrow (-3\cos\alpha)^2+\cos^2\alpha=1 \Rightarrow 10\cos^2\alpha=1 \Rightarrow \cos\alpha=\pm\frac{1}{\sqrt{10}}$ $\xrightarrow{طبق\ log} -\cos\alpha>0,\ \cos\alpha<0$ دلیل $\Leftarrow \frac{-1}{\sqrt{10}}$ ✓

$\sin\alpha-\cos\alpha=-4\cos\alpha=-4(\frac{-1}{\sqrt{10}})=\boxed{\frac{2\sqrt{10}}{5}}$

(۲)

۴

$\sin^2\theta=x,\ \cos^2\theta=y \Rightarrow \underbrace{y+x=1}_{y=1-x} \Rightarrow x^2+y=\frac{4}{5} \Rightarrow x^2(1-x)=\frac{4}{5} \Rightarrow x=\frac{5\pm\sqrt{25-20}}{10}=\frac{5\pm\sqrt{5}}{10}$

$\Rightarrow y^2+x=(1-x)^2+x=1-x+x^2 \Rightarrow If\ x=\frac{5\pm\sqrt{5}}{10} \Rightarrow 1-x+x^2=\frac{3\pm\sqrt{5}}{10} \Rightarrow 1+\frac{-2\pm 2\sqrt{5}}{10}=1-\frac{2}{10}\pm\frac{2\sqrt{5}}{10}$

$=\boxed{\frac{4}{5}\pm\frac{\sqrt{5}}{5}}$

$\sin^4\theta+\cos^2\theta=\frac{4}{5} \to (1-\cos^2\theta)^2+\cos^2\theta=\frac{4}{5} \to \cos^4\theta-\cos^2\theta+1=\frac{4}{5}$

$\cos^4\theta-1+\sin^2\theta+1=\frac{4}{5} \to \cos^4\theta+\sin^2\theta=\frac{4}{5}$

(۱/۲۵)

۵

$\sin x\tan x=\frac{\sin^2x}{\cos x} \Rightarrow \sin x\tan x-\frac{1}{\cos x}>0 \Rightarrow \frac{\overbrace{\sin^2x-1}^{-\cos^2x}}{\cos x}>0 \Rightarrow -\cos x>0 \Rightarrow \cos x<0$ (۲)

$2-2\cos^2x=2(1-\cos^2x)=2\sin^2x \Rightarrow \frac{3\sin x+\sin x\cos x}{2-2\cos^2x}<0 \Leftrightarrow \frac{\sin x\overbrace{(3+\cos x)}^{+}}{\underbrace{2\sin^2x}_{+}}<0$ دلیل $\sin x<0$ (۱)

$\boxed{ناحیه\ ۳} \Leftarrow (۱),(۲)$

(۲)

لیلی ضیاضیایی

$$\frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \Rightarrow 2\cos x + 3\sin x = 0 \Rightarrow 2\cos x = -3\sin x \Rightarrow \frac{\cos x}{\sin x} = \frac{-2}{3}$$

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{-3}{2} - \frac{2}{3} = \boxed{\frac{-13}{6}}$$

۲ ۶

$$1 + \sin 2\omega = \sin^2 2\omega + \cos^2 2\omega + 2\sin 2\omega \cos 2\omega = (\sin 2\omega + \cos 2\omega)^2$$

$$\sin \omega + \sin 1\omega = 2\sin \frac{\omega + 1\omega}{2} \cos \frac{\omega - 1\omega}{2} = 2\sin \frac{1}{2} \cos 1\omega = \cos 2\omega$$

$$\sin 2\omega + \cos 2\omega = \sqrt{2}(\cos 4\omega \sin 2\omega + \sin 4\omega \cos 2\omega) = \sqrt{2}(\sin(2\omega + 4\omega)) = \sqrt{2}\sin 6\omega$$

$$\sin 6\omega = \cos 2\omega \Rightarrow \frac{\sqrt{2}\cos 2\omega}{\cos 2\omega} = \boxed{\sqrt{2}}$$

$\frac{\sin 2\omega + \cos 2\omega}{\cos 2\omega}$

۲ ۷

$$1 - 2\sin^2 \omega = \cos 1\omega$$

$$\frac{1 - \tan^2 2\omega}{1 + \tan^2 2\omega} = \cos 4\omega$$

$$2\cos^2 1\omega - 1 = \cos 2\omega$$

$$\cos 1\omega \cdot \cos 2\omega \cdot \cos 4\omega \times \frac{2\cos 1\omega}{2\cos 1\omega} = \frac{2\sin 4\omega \cos 4\omega}{2 \times 4 \sin 1\omega} = \frac{\sin 8\omega}{8 \sin 1\omega}$$

$$= \frac{\cos 1\omega}{8 \sin 1\omega} = \boxed{\frac{1}{8}\cot(1\omega)}$$

۲ ۸

$$\sin \alpha = 2 - 3\cos \alpha \Rightarrow (2 - 3\cos \alpha)^2 + \cos^2 \alpha = 1 \Rightarrow 10\cos^2 \alpha - 12\cos \alpha + 3 = 0 \Rightarrow \cos \alpha =$$

$$\frac{12 \pm \sqrt{144 - 120}}{20} = \frac{12 \pm \sqrt{24}}{20} = \frac{6 \pm \sqrt{6}}{10}$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1 = 2\left(\frac{6 \pm \sqrt{6}}{10}\right)^2 - 1 = 2\left(\frac{42 \pm 12\sqrt{6}}{100}\right) - 1 = \frac{-4 \pm 6\sqrt{6}}{20}$$

$$\omega \cos 2\alpha - 12\cos \alpha = \omega\left(\frac{-4 \pm 6\sqrt{6}}{20}\right) - 12\left(\frac{6 \pm \sqrt{6}}{10}\right) = \frac{-4 \pm 6\sqrt{6} - 36 \mp 6\sqrt{6}}{\omega} = \frac{-40}{\omega} = \boxed{-8}$$

۲ ۹

$$\sin B \cos C = \cos^2 \frac{A}{2} \Rightarrow \sin 72 \sin(108 - A) = \cos^2 \frac{A}{2} \Rightarrow \sin 72(\sin 72 + A) = \cos^2 \frac{A}{2}$$

$$A + 72 + C = 180 \Rightarrow C = 108 - A$$

$$\Rightarrow \sin 72 \sin(72 + A) = \frac{1 + \cos A}{2} \quad \text{If } A = 36 \Rightarrow \sin 72 \sin 108 = \sin^2 72 = \cos^2 18$$

$$\boxed{A = 36}$$

۲ ۱۰