۱ـ $S=\frac{\text{قاعده}\times\text{ارتفاع}}{2}$ ، قاعده $=T=\frac{2\pi}{|-2|}=\pi$

ارتفاع: $x=0 \to \tan(-2(0)+\frac{\pi}{4})=+1$ $\Big\} \to S=\frac{1\times\pi}{2}=\frac{\pi}{2}$

۲ـ $x=\frac{2}{3} \to \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}(1-\sin^2\alpha)=0$

$=\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36}=0 \to \begin{cases}\sin\alpha=\frac{7}{3} & \times\\ \sin\alpha=\frac{1}{3} & \checkmark\end{cases}$

$\Rightarrow \sin^2\alpha+\cos^2\alpha=1 \to \cos^2\alpha=\frac{8}{9} \to 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=\frac{9}{8}$

۳ـ $\log(\sin\alpha)-\log(-\cos\alpha)=\log\frac{\sin\alpha}{-\cos\alpha}=\log(-\tan\alpha)=\log 3$

$\Rightarrow \tan\alpha=-3 \Rightarrow (\sin\alpha-\cos\alpha)^2=1-2\sin\alpha\cos\alpha$

$=1-2\left(\frac{1}{\tan\alpha+\cot\alpha}\right)=1-2\left(\frac{1}{-3-\frac{1}{3}}\right)=1+\frac{3}{5}=\frac{8}{5}$

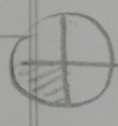
$\xrightarrow{\text{جذر}} \sin-\cos=\pm\sqrt{\frac{8}{5}} \xrightarrow{\text{ربع ۲}} +\sqrt{\frac{8}{5}}$

۴ـ $\sin^4\theta+1-\sin^2\theta=\frac{4}{5} \Rightarrow \sin^4\theta-\sin^2\theta=-\frac{1}{5}$

$\cos^4\theta+\sin^2\theta=(1-\sin^2\theta)^2+\sin^2\theta=\underbrace{\sin^4\theta-\sin^2\theta}_{-\frac{1}{5}}+1=\frac{4}{5}$

۵ـ $\frac{\sin x(3+\cos x)}{2(\underbrace{1-\cos^2x}_{\sin^2x})}=\frac{\overset{\text{همواره }+}{3+\cos x}}{2\sin x}<0 \Rightarrow \sin x<0$

$\frac{\sin^2x}{\cos x}-\frac{1}{\cos x}>0 \to \frac{-\cos^2x}{\cos x}>0 \Rightarrow \cos x<0$



ربع ۳

$$\frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \longrightarrow 2\cos x = -3\sin x$$ — 6

$$\Longrightarrow \tan x = -\frac{2}{3}, \quad \cot x = -\frac{3}{2} \longrightarrow \tan x + \cot x = -\frac{13}{6}$$

$$\sin 50 + 1 = (\sin 25 + \cos 25)^2 \longrightarrow \sqrt{\sin 50 + 1} = \sin 25 + \cos 25$$ — 7

$$\sin 10 + \sin 50 = 2\sin 30 \cos 20 = \cos 20 \Longrightarrow A = \frac{\sin 25 + \cos 25}{\cos 20} = \frac{\sqrt{2}\sin(25+45)}{\cos 20}$$

$$\frac{\sqrt{2}\sin 70}{\sin 70} = \sqrt{2}$$

$$\frac{\sin 10 (\cos 10^\circ)(\cos 20^\circ)(\cos 40^\circ)}{\sin 10 \times 2 \times 2 \times 2} = \frac{\sin 80}{\sin 10 \times 8} = \frac{\cos 10}{\sin 10 \times 8}$$ — 8

$$= \cot 10^\circ \times \frac{1}{8}$$

— 9

$$\sqrt{10}\sin(\alpha + x) = 2 \longrightarrow \sin(\alpha + x) = \frac{\sqrt{10}}{5}$$

$$\cos(\alpha + x) = \sqrt{1 - \sin^2(\alpha + x)} = \frac{\sqrt{15}}{5}$$

$$\cos(\alpha + x) = \cos\alpha\cos x - \sin\alpha \sin x \Longrightarrow \frac{\sqrt{15}}{5} = \frac{1}{\sqrt{10}}(\cos\alpha - 3\sin\alpha)$$

$$\longrightarrow \begin{cases} \cos\alpha - 3\sin\alpha = \sqrt{6} \\ \sin\alpha + 3\cos\alpha = 2 \end{cases} \Longrightarrow \sin\alpha = \frac{2 - 3\sqrt{6}}{10}, \quad \cos\alpha = \frac{\sqrt{6} + 6}{10}$$

$$5\cos 2\alpha = 5(2\cos^2\alpha - 1) \longrightarrow \cos 2\alpha = \frac{-4 + 6\sqrt{6}}{5}$$

$$12\cos\alpha = \frac{6\sqrt{6} + 36}{5}$$

$$5\cos 2\alpha + 12\cos\alpha = \frac{-36 - 6\sqrt{6}}{5} + \frac{-4 + 6\sqrt{6}}{5} = \frac{-40}{5}$$ — 8

SCbo

$$\hat{A} + ۷۲ + \hat{C} = ۱۸۰ \Rightarrow \hat{C} = ۱۰۸ - A \qquad -۱۰$$

$$۲\sin ۷۲ \times (\sin ۱۰۸ - A) = ۱ + \cos A \,, \quad \sin ۱۰۸ = \sin ۷۲ \,, \quad \cos ۱۰۸ = -\cos ۷۲$$

$$\Rightarrow ۲\sin(۷۲) \times (\sin ۷۲ \cos A + \sin A \cos ۷۲) = ۱ + \cos A$$

$$\Rightarrow ۲\sin^{۲} ۷۲ \cos A + ۲\sin ۷۲ \cos ۷۲ \sin A = ۱ + \cos A$$

$$(۲\sin^{۲} ۷۲ - ۱)(\cos A + ۲\sin ۷۲ \cos ۷۲ \sin A = ۱$$

$$-\cos ۱۴۴ \times \cos A + \sin ۱۴۴ \times \sin A = ۱ = \cos(A - ۱۴۴)$$

$$\Rightarrow \cos(A - ۱۴۴) = \underbrace{۱}_{۲k\pi} \longrightarrow A - ۱۴۴ = ۲k\pi \Rightarrow A = ۳۶^{\circ}$$