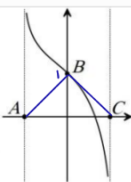


۲۰ آفرین



$$y = \tan(-2x + \frac{\pi}{4}) \xrightarrow{x=0} y = \tan \frac{\pi}{4} = 1$$

$$\left. \begin{aligned} A: -2x + \frac{\pi}{4} &= \frac{\pi}{4} \Rightarrow x = -\frac{\pi}{8} \\ B: -2x + \frac{\pi}{4} &= -\frac{\pi}{4} \Rightarrow x = \frac{3\pi}{8} \end{aligned} \right\} \Rightarrow C - A = \frac{\pi}{2}$$

$$\Rightarrow S_{ABC} = \frac{1}{2} \times 1 \times \frac{\pi}{2} = \frac{\pi}{4}$$

سوال (۱) ۲۰

$$x^2 - (\sin \alpha)x - \frac{1}{4} \cos^2 \alpha = 0 \xrightarrow{x=\frac{1}{2}} \frac{1}{4} - \frac{1}{4} \sin \alpha - \frac{1}{4} \cos^2 \alpha = 0 \rightarrow \frac{1}{4} - \frac{1}{4} \sin \alpha - \frac{1}{4} (1 - \sin^2 \alpha) = 0$$

$$\frac{1}{4} \sin^2 \alpha - \frac{1}{4} \sin \alpha + \frac{1}{4} = 0 \xrightarrow{\times 4} \sin^2 \alpha - \sin \alpha + 1 = 0 \rightarrow (3 \sin \alpha - 1)(3 \sin \alpha - 4) = 0 \rightarrow \sin \alpha = \frac{1}{3} \vee \sin \alpha = \frac{4}{3}$$

$$\sin^2 \alpha = \frac{1}{9} \rightarrow \cos^2 \alpha = 1 - \frac{1}{9} = \frac{8}{9} \rightarrow 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} = \frac{1}{\frac{8}{9}} = \frac{9}{8}$$

سوال (۲) ۲۰

$$\log(\sin \alpha) - \log(-\cos \alpha) = \log 3 \rightarrow \log\left(\frac{\sin \alpha}{-\cos \alpha}\right) = \log 3 \rightarrow \log(-\tan \alpha) = \log 3 \rightarrow \tan \alpha = -3$$



از اینجا می‌توانیم $\tan \alpha$ است پس یا $\cos \alpha < 0$ یا $\sin \alpha < 0$ از اینجا می‌توانیم که یکی صورت سوال $\cos \alpha$ یا $\sin \alpha$ است پس می‌توانیم $\cos \alpha$ برداشت تا کاریم تعریف شود

$$\rightarrow \sin \alpha = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}, \cos \alpha = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10} \Rightarrow \sin \alpha - \cos \alpha = \frac{3\sqrt{10}}{10} + \frac{\sqrt{10}}{10} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$$

سوال (۳) ۲۰

$$\sin^2 \theta + \cos^2 \theta = \frac{4}{9} \rightarrow \sin^2 \theta - \sin^2 \theta + 1 = \frac{4}{9} \rightarrow \sin^2 \theta (\sin^2 \theta - 1) = -\frac{1}{9} \Rightarrow \sin^2 \theta \cos^2 \theta = \frac{1}{9}$$

$$\cos^2 \theta + \sin^2 \theta - \cos^2 \theta - \cos^2 \theta + 1 = \cos^2 \theta (\cos^2 \theta - 1) + 1 = -\sin^2 \theta \cos^2 \theta + 1 = -\frac{1}{9} + 1 = \frac{8}{9}$$

سوال (۴) ۲۰

$$\frac{3 \sin n + \sin n \cos n}{2 - 2 \cos^2 n} \rightarrow \frac{\sin n (3 + \cos n)}{2 - 2(1 - \sin^2 n)} \rightarrow \frac{\sin n (3 + \cos n)}{2 - 2 + 2 \sin^2 n} \rightarrow \frac{3 + \cos n}{2 \sin n} \rightarrow \sin n$$

سوال (۵) ۲۰

$$\sin n \tan n - \frac{1}{\cos n} \rightarrow \frac{\sin^2 n}{\cos n} - \frac{1}{\cos n} \rightarrow \frac{\sin^2 n - 1}{\cos n} \rightarrow \frac{-\cos^2 n}{\cos n} \rightarrow \cos n$$

نصیه دوم برابر

$$\frac{2}{\sin n} + \frac{3}{\cos n} = 0 \Rightarrow \frac{2 \cos n + 3 \sin n}{\sin n \cos n} = 0 \rightarrow 2 \cos n = -3 \sin n \rightarrow \tan n = -\frac{2}{3}, \cot n = -\frac{3}{2}$$

سوال (۶) ۲۰

$$\tan n + \cot n = -\frac{2}{3} - \frac{3}{2} = \frac{-4-9}{6} = -\frac{13}{6}$$

$$\frac{\sqrt{1 + \sin \alpha}}{\sin \alpha + \sin \alpha} = \frac{\sqrt{1 + \cos \alpha}}{\sin(\alpha + \alpha) + \sin(\alpha - \alpha)} = \frac{\sqrt{2 \cos \frac{\alpha}{2}}}{\sin \alpha \cdot \cos \frac{\alpha}{2} + \sin \alpha \cdot \cos \frac{\alpha}{2} + \sin \alpha \cdot \cos \frac{\alpha}{2} - \sin \alpha \cdot \cos \frac{\alpha}{2}} = \frac{\sqrt{2 \cos \frac{\alpha}{2}}}{2 \sin \alpha \cdot \cos \frac{\alpha}{2}} = \frac{\sqrt{2 \cos \frac{\alpha}{2}}}{2 \cos \frac{\alpha}{2}} = \sqrt{2}$$

سوال (۷) ۲۰

$$\frac{\cos \frac{\alpha}{2}}{(1 - 2 \sin^2 \frac{\alpha}{2})} \cdot \frac{\cos \frac{\alpha}{2}}{(2 \cos^2 \frac{\alpha}{2} - 1)} \cdot \frac{\cos \frac{\alpha}{2}}{(1 - \tan^2 \frac{\alpha}{2})} = \frac{\sin \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} \cdot \frac{\sin \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} \cdot \frac{\sin \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} = \frac{\sin \frac{\alpha}{2} \times \cos \frac{\alpha}{2} \times \cos \frac{\alpha}{2} \times \cos \frac{\alpha}{2} \times \frac{1}{\sin \frac{\alpha}{2}}}{\frac{1}{2} \sin \frac{\alpha}{2}} = \frac{1}{\frac{1}{2}} \times \frac{\sin \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} = \frac{1}{\frac{1}{2}} \times \frac{\cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} = \frac{1}{\frac{1}{2}} \cot \frac{\alpha}{2}$$

سوال (۸) ۲۰

$$\sin \alpha + 12 \cos \alpha = 13 \rightarrow \sin \alpha = 13 - 12 \cos \alpha \xrightarrow{\text{قوسین}} \sin^2 \alpha = 169 - 24 \cos \alpha + 144 \cos^2 \alpha \xrightarrow{\sin^2 \alpha = 1 - \cos^2 \alpha} \text{سوال ۹}$$

$$1 - \cos^2 \alpha - 144 \cos^2 \alpha + 24 \cos \alpha = 0 \Rightarrow 1 - 145 \cos^2 \alpha + 24 \cos \alpha = 0$$

$$145 \cos^2 \alpha - 24 \cos \alpha - 1 = 0 \Rightarrow (13 \cos \alpha - 1)(11 \cos \alpha + 1) = 0 \Rightarrow 13 \cos \alpha = 1 \Rightarrow \cos \alpha = \frac{1}{13}$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{1}{169}} = \sqrt{\frac{168}{169}} = \frac{\sqrt{168}}{13}$$

$$\sin \hat{B} \sin \hat{C} = \cos \frac{A}{2} \xrightarrow{\cos \frac{A}{2} = \frac{1 + \cos A}{2}} 2 \sin \hat{B} \sin \hat{C} = 1 + \cos \hat{A}$$

$$\cos(\pi - (B+C)) = -\cos(B+C)$$

$$2 \sin B \sin C = 1 - \cos(B+C) \Rightarrow 2 \sin B \sin C = 1 - \cos B \cos C + \sin B \sin C \Rightarrow \cos B \cos C + \sin B \sin C = 1$$

$$\Rightarrow \cos(B-C) = 1 \Rightarrow B-C = 0 \Rightarrow B = C = 47 \Rightarrow A = 180 - 2(47) = 86 \Rightarrow \hat{A} = 86^\circ$$