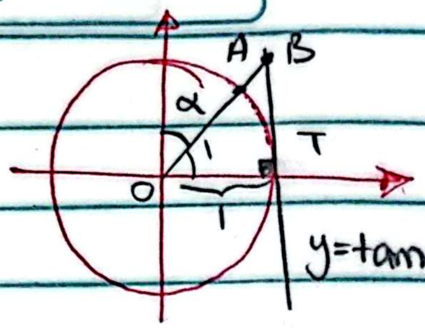
حل تمرین‌ها

Subject :   Date : / /



$TB = \tan O_1 = \cot\alpha$ ☆ (1)

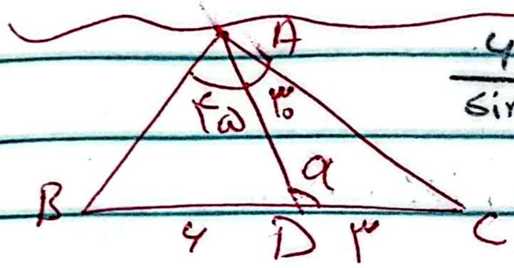
$y = \tan x$ $OT^2 + TB^2 = OB^2 \Rightarrow$

$1 + \cot^2\alpha = OB^2 \rightarrow$

$OB = \sqrt{1+\cot^2\alpha} \rightarrow OB = \sqrt{\frac{1}{\sin^2}} \xrightarrow[\text{اول}]{\text{ربع}} OB = \frac{1}{\sin\alpha}$

$OA = OT = 1 \rightarrow OB = OA + AB \Rightarrow \frac{1}{\sin\alpha} = 1 + AB \Rightarrow$

$\frac{1}{\sin\alpha} - 1 = \frac{-\sin\alpha + 1}{\sin\alpha}$



$\frac{6}{\sin 45} = \frac{AD}{\sin B} = \frac{AB}{\sin D}$; $\frac{AB}{AC} = ?$ (2)

$\frac{3}{\sin 30} = \frac{AD}{\sin C} = \frac{AC}{\sin\alpha} \Rightarrow$

$\frac{6}{\frac{\sqrt2}{2}} = \frac{AD}{\sin B} \Rightarrow \frac{12\times\sqrt2}{2} = 6\sqrt2 = \frac{AD}{\sin B} \rightarrow 6\sqrt2\sin\hat B = AD$ (1)

$\frac{6}{\frac12} = \frac{AD}{\sin C} \Rightarrow 12\sin\hat C = AD$ (2) $\xrightarrow{(2),(1)} 12\sin\hat C = 6\sqrt2\sin\hat B$

$\rightarrow 2\sin\hat C = \sqrt2\sin\hat B \Rightarrow \frac{\sqrt2}{2}\sin\hat B = \sin\hat C$ ☆

$\frac{AC}{\sin B} = \frac{AB}{\sin C}$ ☆ →   قضیه سینوس در مثلث بزرگ ←

$\frac{AB}{AC} = \frac{\sin\hat C}{\sin B} \Rightarrow \frac{\frac{\sqrt2}{2}\sin\hat B}{\sin\hat B} = \frac{\sqrt2}{2}$

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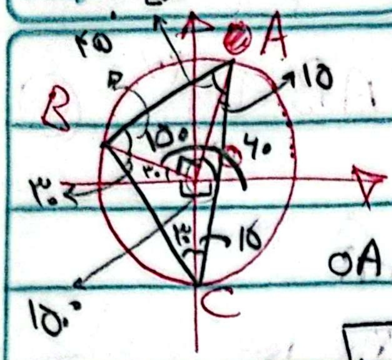

۳) طول A ← $\frac{1}{2}$ عرض B ← $\frac{1}{2}$

محیط ABC ؟

$OA = OB = OC = R = 1$

$OA = \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2} \Rightarrow 1 = \sqrt{(\frac{1}{2} - 0)^2 + (y_A - 0)^2} = 1$

$\frac{1}{4} + (y_A)^2 = 1 \rightarrow y_A^2 = \frac{3}{4} \rightarrow y_A = \frac{\sqrt{3}}{2} \rightarrow A \left| \begin{matrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{matrix} \right. \rightarrow 60°$

$OB = \sqrt{(x_B - x_0)^2 + (y_B - y_0)^2} \Rightarrow 1 = \sqrt{x_B^2 + \frac{1}{4}} \Rightarrow 1 = \frac{1}{4} + x_B^2 \Rightarrow$

$x_B = \frac{-\sqrt{3}}{2} \quad B = \left| \begin{matrix} \frac{-\sqrt{3}}{2} \\ \frac{1}{2} \end{matrix} \right. \quad C \left| \begin{matrix} 0 \\ -1 \end{matrix} \right.$

$AB = \sqrt{(x_A - x_B)^2 - (y_A - y_B)^2} = \sqrt{(\frac{1}{2} + \frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2} - \frac{1}{2})^2} \Rightarrow$

$\sqrt{\frac{1}{4} + \frac{3}{4} + \frac{\sqrt{3}}{2} + \frac{3}{4} + \frac{1}{4} - \frac{\sqrt{3}}{2}} = \sqrt{2}$

$AC = \sqrt{(0 - \frac{1}{2})^2 + (-1 - \frac{\sqrt{3}}{2})^2} \Rightarrow \sqrt{\frac{1}{4} + 1 + \frac{3}{4} + \sqrt{3}} \Rightarrow \sqrt{2 + \sqrt{3}}$

$AC \times AB \times \sin 60 \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \sqrt{2} \times \sqrt{2 + \sqrt{3}} \times \frac{1}{2} = \frac{\sqrt{12 + 6\sqrt{3}}}{4}$

$\sqrt{\frac{(3 + \sqrt{3})^2}{4}} = \frac{3 + \sqrt{3}}{4}$

$BC = \sqrt{(0 + \frac{\sqrt{3}}{2})^2 + (-1 - \frac{1}{2})^2} \Rightarrow \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$ محیط ABC

$\sqrt{3} + \sqrt{2} + \sqrt{2 + \sqrt{3}} =$ محیط

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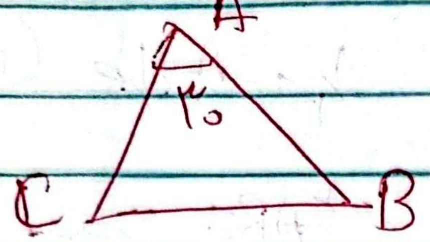

۴) $AC = \log_{۳}^{۱۶}$ $AB = \log_{۲}^{۲۷}$

$\frac{۱}{۲} \times AC \times AB \times \sin \hat{A} =$

$\frac{۱}{۲} \times ۴ \log_{۳}^{۲} \times ۳ \log_{۲}^{۳} \times \frac{۱}{۲} = \frac{۱}{۴} \times ۱۲ \times \cancel{\log_{۳}^{۲}} \times \frac{۱}{\cancel{\log_{۳}^{۲}}} =$ ۳

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۵) $۲x^{۴} - x^{۲} - \sin^{۲}(\alpha)(۲\sin^{۲}\alpha - ۱)$ $x - \cos\alpha$

$x = \cos\alpha$

$۲\cos^{۴}\alpha - \cos^{۲}\alpha - \sin^{۲}\alpha(۲\sin^{۲}\alpha - ۱) = ۱ \longrightarrow$

$۲\cos^{۴}\alpha - \cos^{۲}\alpha - ۲\sin^{۴}\alpha + \sin^{۲}\alpha = ۱ \Rightarrow$

$۲(\cos^{۲}x - \sin^{۲}x)(\cos^{۲}x + \sin^{۲}x) + (\sin^{۲}\alpha - \cos^{۲}\alpha) = ۱ \Rightarrow$

$۲\cos ۲\alpha + \cos ۲\alpha = ۱ \Rightarrow ۳\cos ۲\alpha = ۱ \longrightarrow \cos ۲\alpha = \frac{۱}{۳}$

$۱ - (\cos ۲\alpha)^{۲} = (\sin ۲\alpha)^{۲} \Rightarrow ۱ - \frac{۱}{۹} = \frac{۸}{۹} \longrightarrow$

$\sin ۲\alpha = \frac{۲\sqrt{۲}}{۳}$

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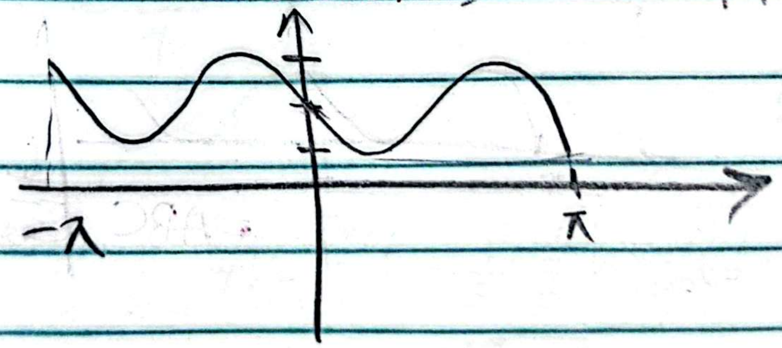
۶) $y = -۲\sin x \cos x + ۲ \longrightarrow -\sin ۲\alpha + ۲ \rightarrow$

$T = \frac{۲\pi}{۲} = \pi$



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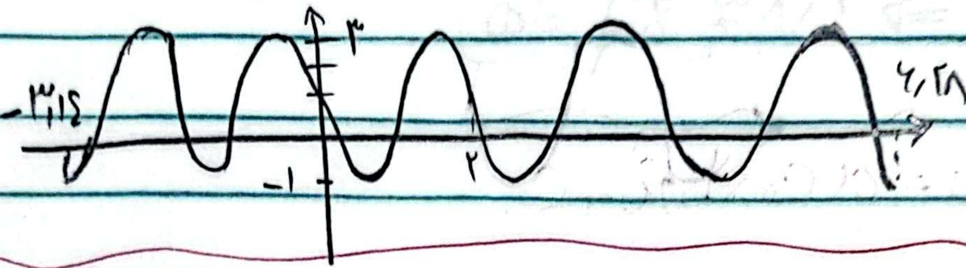
$-3{,}14 \leftarrow [-\pi, 2\pi] \rightarrow 2\times 3{,}14 = 6{,}28$

$y = 2\cos(x + \frac{1}{2})\pi + 1 \Longrightarrow$

$y = 2\cos(x\pi + \frac{\pi}{2}) + 1 \qquad T = \frac{2\pi}{\pi} = 2$



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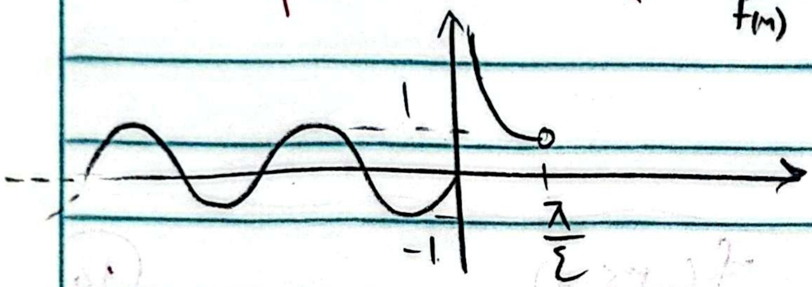
$$f(x) = \begin{cases} \cot x & 0 < x < \frac{\pi}{4} \\ \sin x & x \leq 0 \end{cases} \qquad R_{f(x)} = [-1, +\infty)$$



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$f(x) = a + b\sin(cx)\cos(cx)\cos(2cx)$

$a + b\,\frac{\sin(2cx)}{2} \times \cos(2cx) \Longrightarrow$

$f(x) = a + b\,\frac{\sin(4cx)}{4} = a + \frac{b}{4}\sin(4cx)$

$T = \frac{2\pi}{b} = \frac{4\pi}{10} \Longrightarrow \frac{2\pi \times 10}{4\pi} = 5$

$5 = 4c \Longrightarrow c = \frac{5}{4}$

$a = \frac{۴+۰}{۲} = ۲ \qquad b' = \frac{۴}{۲} = ۲' \longrightarrow \frac{b-۰}{۴} = ۲ \Rightarrow b = ۸$

$\frac{bc}{a} \Rightarrow ۸ \times \frac{۵}{۴} \times \frac{۱}{۲} = ۵$

$f(x) = a\cos bx + c$ (۹)

$a = \frac{۲+۴}{۲} = \frac{۶}{۲} = ۳ \qquad c = \frac{-۴+۲}{۲} = \frac{-۲}{۲} = -۱ \qquad T = \frac{۲\pi}{b} = \pi \Rightarrow b = ۲$

باید مثبت باشه

$-۳\cos ۲x - ۱ = ۰ \longrightarrow \cos ۲x = \frac{-۱}{۳} \xrightarrow{x=\alpha} \cos ۲\alpha = \frac{-۱}{۳}$

$\longrightarrow |\sin ۲\alpha| = \sqrt{۱ - \frac{۱}{۹}} = \frac{۲\sqrt{۲}}{۳}$

$|\tan ۲\alpha| = \left|\frac{\sin ۲\alpha}{\cos ۲\alpha}\right| = ۲\sqrt{۲}$

$y = f(x) \qquad T = ۳ \qquad f(-۳۴/۵)$ (۱۰)

$-۳۴/۵ = -k \times ۳۶ + ۱/۵ \longrightarrow f(-۳۴/۵) = f(۰/۵)$

$-۱ + ۱/۵ = ۰/۵$

$$f(x) = \begin{cases} \frac{-۱}{۲}x + ۱ & ۰ \leq x < ۲ \\ ۲x + ۲ & -۱ \leq x < ۰ \end{cases}$$

$f(۰/۵) = \frac{-۱}{۲} \times ۰/۵ + ۱ = ۰/۷۵$