مبینا جعفری پور — تکلیف ۱۴ — دوازدهم دختر B

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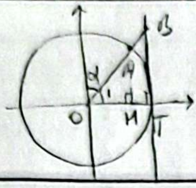

$O_1 = 90 - \alpha$

$AH = \sin O_1 = \sin(90-\alpha) = \cos\alpha$

$BT = \tan O_1 = \tan(90-\alpha) = \cot\alpha$

$AH \parallel BT \xRightarrow{\text{نتیجه تالس}} \frac{AH}{BT} = \frac{OA}{OB} \Rightarrow \frac{\cos\alpha}{\cot\alpha} = \frac{1}{OB} \Rightarrow OB = \frac{\cot\alpha}{\cos\alpha} = \frac{1}{\sin\alpha}$

$\Rightarrow AB = OB - OA = \boxed{\frac{1}{\sin\alpha} - 1}$

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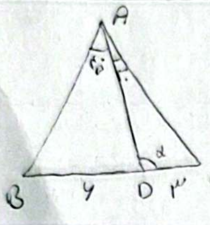

$\frac{S_{\hat{ACD}}}{S_{\hat{ABD}}} = \frac{CD}{BD} = \frac{3}{6} = \frac{AC \times AD \times \frac{1}{2} \times \sin 30^\circ}{AB \times AD \times \frac{1}{2} \times \sin 45^\circ} = \frac{AC \times \frac{1}{2}}{AB \times \frac{\sqrt{2}}{2}} \Rightarrow \boxed{\frac{AB}{AC} = \sqrt{2}}$

هم رأس هستند. پس ارتفاع برابر دارند و نسبت مساحت ها برابر نسبت قاعده ها است

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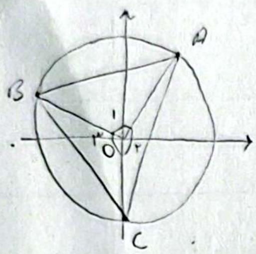

$x_A = \frac{1}{2} \xRightarrow{\text{ناحیه اول}} \hat{A} = 60^\circ$

$y_B = \frac{1}{2} \xRightarrow{\text{ناحیه دوم}} \hat{B} = 150^\circ$

$\Rightarrow \begin{cases} \hat{O}_1 = \hat{B} - \hat{A} = 90^\circ \Rightarrow S_{\hat{AOB}} = \frac{OA \times OB}{2} = \frac{1}{2} \\ \hat{O}_2 = \hat{A} + 90^\circ = 150^\circ \Rightarrow S_{\hat{AOC}} = \frac{OA \times OC}{2} \times \sin 150^\circ = \frac{1}{4} \\ \hat{O}_3 = 90 + 180 - \hat{B} = 120^\circ \Rightarrow S_{\hat{BOC}} = \frac{OB \times OC \times \sin 120^\circ}{2} = \frac{\sqrt{3}}{4} \end{cases} \Rightarrow \boxed{S_{\hat{ABC}} = \frac{3+\sqrt{3}}{4}}$

$AB = \sqrt{OA^2 + OB^2 - 2OA.OB.\cos O_1} = \sqrt{2}$, $\cos 90^\circ \to 0$

$BC = \sqrt{OC^2 + OB^2 - 2OC.OB.\cos O_3} = \sqrt{3}$, $\cos 120^\circ = -\frac{1}{2}$

$AC = \sqrt{OA^2 + OC^2 - 2OC.OB.\cos O_2} = \sqrt{2+\sqrt{3}}$, $\cos 150^\circ = -\frac{\sqrt{3}}{2}$

$\boxed{P_{ABC} = \sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}}$

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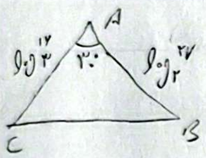

$S_{ABC} = \frac{1}{2} AB \times AC \times \sin\hat{A} = \frac{1}{2} \times \log_2 27 \times \log_3 16 \times \sin 30^\circ = \frac{1}{4}(3\log_2 3 \times 4\log_3 2) = 3 \log_2 3 \cdot \log_3 2 = \boxed{3}$

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$x - \cos\alpha = 0 \Rightarrow \cos\alpha = x \Rightarrow 2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = \cos^2\alpha(\underbrace{2\cos^2\alpha - 1}_{\cos 2\alpha}) - \sin^2\alpha(-\cos 2\alpha) = \cos 2\alpha(\underbrace{\cos^2\alpha + \sin^2\alpha}_{1})$

$= \cos 2\alpha = 1 \Rightarrow \sin 2\alpha = \sqrt{1 - \cos^2 2\alpha} = \boxed{0}$

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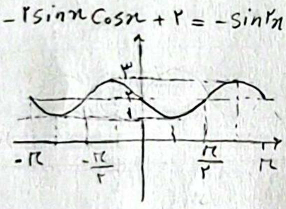
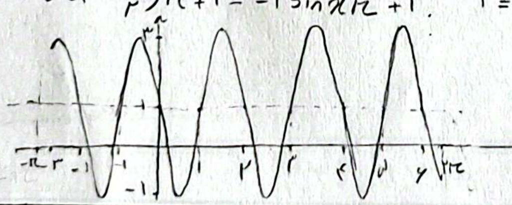
الف) $y = -2\sin x\cos x + 2 = -\sin 2x + 2$ $\quad T = \frac{2\pi}{2} = \pi \quad [-\pi, \pi] \quad max = |-1| + 2 = 3 \quad min = -|-1| + 2 = 1$



ب) $y = 2\cos(x + \frac{1}{2})\pi + 1 = -2\sin x\pi + 1$ $\quad T = \frac{2\pi}{\pi} = 2 \quad [-\pi, 2\pi] \quad max = |-2| + 1 = 3 \quad min = -|-2| + 1 = -1$



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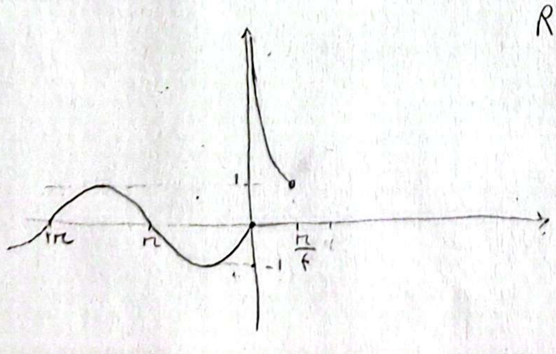
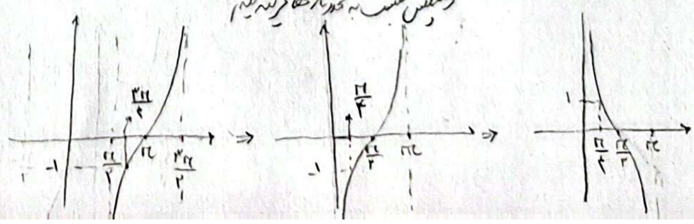
$R_f = [-1, +\infty)$



$\cot x = -\tan(x + \frac{\pi}{2})$ کافی است نمودار tan را $\frac{\pi}{2}$ به چپ انتقال دهیم سپس نسبت به محور x ها قرینه کنیم



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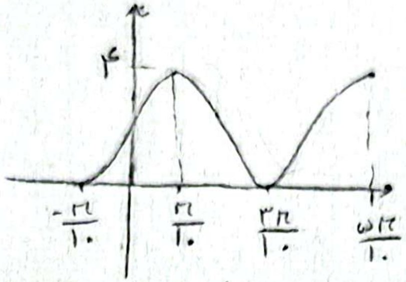
$$f(x) = a + \underbrace{b\sin(cx)\cos(cx)}_{\frac{b}{2}\sin(2cx)}\cos(2cx) = a + \frac{b}{4}\sin 4cx$$

$$T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{2\pi}{5} = \frac{2\pi}{4|c|} \Rightarrow |c| = \frac{5}{4} \Rightarrow c = \frac{5}{4}$$

$$\left.\begin{aligned} max &= 4 = a + \frac{|b|}{4} \\ min &= 0 = a - \frac{|b|}{4} \end{aligned}\right\} \Rightarrow a = 2 ,\ |b| = 8 \Rightarrow b = 8$$



با توجه به شکل تابع متوجه می‌شیم که در همسایگی $x=0$ تابع صعودی است

پس $bc>0$ و از آنجایی که طبق فرض $c>0$ است پس

$b>0$ می‌شود

البته با ترکیدن نقطهٔ $\frac{\pi}{2}$ هم متوجه می‌شدیم:

$$f\left(\frac{\pi}{2}\right) = 2 + \frac{b}{4}\sin\left(5\frac{\pi}{2}\right) = 2 + \frac{b}{4} = 4 \Rightarrow b = 8$$

$$\Rightarrow \frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = \boxed{5}$$

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$$T = \pi = \frac{2\pi}{|b|} \Rightarrow |b| = 2$$

تابع در همسایگی $x=0$ صعودی $\Rightarrow a<0 \Rightarrow a=-3$

$$\left.\begin{aligned} max &= 2 = |a| + c \\ min &= -4 = -|a| + c \end{aligned}\right\} \Rightarrow |a| = 3 ,\ c = -1$$

$$f(x) = -3\cos 2x - 1 \Rightarrow f(\alpha) = -3\cos 2\alpha - 1 = 0 \Rightarrow \cos 2\alpha = -\frac{1}{3}$$

$$|\tan 2\alpha| = \sqrt{\frac{1}{\cos^2 2\alpha} - 1} = \boxed{2\sqrt{2}}$$

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$T>0$ دوره تناوب $\Rightarrow f(x) = f(x \pm T) \Rightarrow f(-34.5) = f(+1.5 - 12(3)) = f(+1.5)$

معادله خط بازهٔ ۰ تا ۲ : $\begin{matrix}(0,1)\\(2,0)\end{matrix} \Rightarrow m = \frac{0-1}{2-0} = -\frac{1}{2}$

$$y - 0 = -\frac{1}{2}(x-2) \Rightarrow y = -\frac{1}{2}x + 1 \Longrightarrow \boxed{f(1.5) = \frac{1}{4}}$$