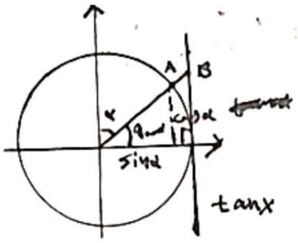
نام و نام خانوادگی ... جعفری — پاسخنامه مثلثات شماره ۱۴۰۱ ... کلاس ...

۱

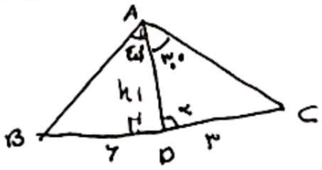


$$\frac{1}{1+AB}=\frac{\cos\alpha}{\cot\alpha}$$

$$\frac{1}{1+AB}=\sin\alpha$$

$$1+AB=\frac{1}{\sin\alpha} \Rightarrow AB=\frac{1}{\sin\alpha}-1$$

۲



$$\frac{AB}{AC}=?$$

$S=\frac{1}{2}ah$

$$\frac{S_{A\hat{D}C}}{S_{A\hat{D}B}}=\frac{1}{2}=\frac{\frac{1}{2}\times ۳\times h}{\frac{1}{2}\times ۶\times h}$$

$$S_{A\hat{D}C}=\frac{1}{2}AD\cdot AC\sin ۳۰^\circ$$

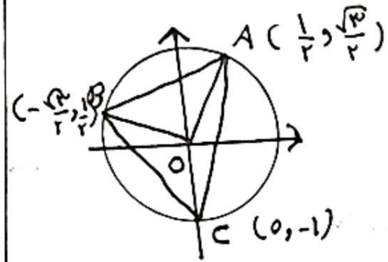
$$S_{A\hat{D}B}=\frac{1}{2}AD\cdot AB\sin ۴۵^\circ$$

$$\Rightarrow \frac{S_{A\hat{D}C}}{S_{A\hat{D}B}}=\frac{AC}{AB}\times\frac{\sin ۳۰^\circ}{\sin ۴۵^\circ}$$

$$=\frac{1}{2}\times\frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}}=\frac{\sqrt{2}}{2}=\frac{AC}{BA}$$

$$\Rightarrow \frac{AB}{AC}=\sqrt{2}$$

۳



الف) $S_{ABC}=?$

$$S_{A\hat{B}C}=\frac{1}{2}\begin{vmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2} & 1\\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 1\\ 0 & -1 & 1\end{vmatrix}=\frac{\sqrt{3}+3}{4}$$

ب)

$$AC^2=(0-\frac{1}{2})^2+(\frac{\sqrt{3}}{2}+1)^2=2+\sqrt{3}\Rightarrow AC=\sqrt{2+\sqrt{3}}$$

$$AB^2=(\frac{1}{2}+\frac{\sqrt{3}}{2})^2+(\frac{\sqrt{3}}{2}-\frac{1}{2})^2=2\Rightarrow AB=\sqrt{2}$$

$$BC^2=(0+\frac{\sqrt{3}}{2})^2+(-1-\frac{1}{2})^2=3\Rightarrow BC=\sqrt{3}$$

$$\boxed{AB+BC+AC=\sqrt{2+\sqrt{3}}+\sqrt{2}+\sqrt{3}}$$

۴

$$AC=\log_3^{16}\qquad AB=\log_2^{27}$$

$$S_{A\hat{B}C}=\frac{1}{2}AB.AC\sin ۳۰^\circ=\frac{1}{2}(\log_2^{27})(\log_3^{16})\times\frac{1}{2}$$

$$=\frac{1}{2}\left(\frac{\log 27}{\log 2}\times\frac{\log 16}{\log 3}\right)\frac{1}{2}=\boxed{3}$$

۵

$$2x^4-x^2-\sin^2\alpha(2\sin^2\alpha-1)\qquad x-\cos\alpha=0\Rightarrow x=\cos\alpha$$

$$\underbrace{2\cos^4\alpha-\cos^2\alpha-2\sin^4\alpha-\sin^2\alpha}=2\cos2\alpha-1=1\Rightarrow\cos2\alpha=1$$

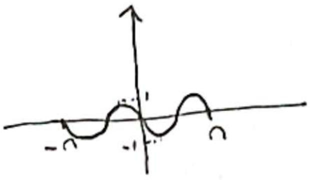
$2\cos2\alpha \qquad -1$

$$\Rightarrow\boxed{\sin2\alpha=0}$$

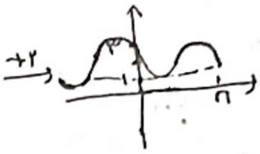
الف) $y=-۲\sin x\cos x+۲ \quad [-\pi,\pi]$

$\Rightarrow y=-\sin ۲x+۲$

$T=\frac{۲\pi}{۲}=\pi$ ۶



$y=-\sin x$



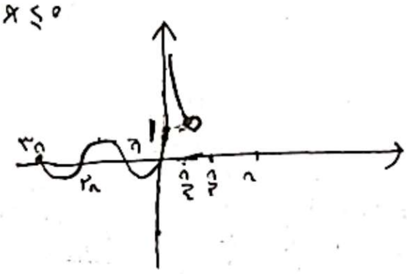
$y=-\sin ۲x+۲$

ب) $y=۲\cos(x+\frac{۱}{۲})\pi+۱=۲\cos(\pi x+\frac{\pi}{۲})+۱=-۲\sin\pi x+۱ \quad [-\pi,۲\pi]$

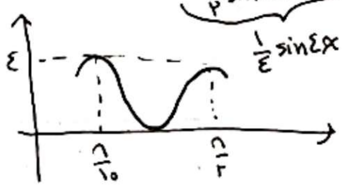
$T=\frac{۲\pi}{\pi}=۲$

$f(x)=\begin{cases}\cot x & ; \ ۰<x<\frac{\pi}{۴}\\ \sin x & ; \ x\leq ۰\end{cases}$ برد تابع؟ ۷

$R_f=[-۱,+\infty)$



$f(x)=a+b\underbrace{\sin(cx)\cos(cx)}_{\frac{۱}{۲}\sin ۲cx}\cos(۲cx)$ $\quad \frac{bc}{a}=?$ $\quad f(x)=a+\frac{b}{۴}\sin ۴cx$

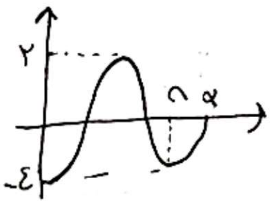


$\frac{۱}{۴}\sin ۴x$

$T=\frac{\pi}{۲}-\frac{\pi}{۱۰}=\frac{۲\pi}{۵}$ $\quad T=\frac{۲\pi}{|۴c|}=\frac{۲\pi}{۵}\Rightarrow |c|=\frac{۵}{۴}\xrightarrow{c>0} c=\frac{۵}{۴}$ ۸

$R_f=[۰,۴]$ $\quad a+\frac{b}{۴}\sin\frac{\pi}{۲}=a+\frac{b}{۴}=۴$ $\quad \frac{bc}{a}=\frac{۸(\frac{۵}{۴})}{۲}=\boxed{۵}$

$a=۲$ $\quad \frac{|b|}{۴}=۲\Rightarrow |b|=۸\Rightarrow b=۸$



$f(x)=a\cos bx+c$ $\quad |\tan ۲\alpha|=?$

$T=\pi=\frac{۲\pi}{|b|}\Rightarrow |b|=۲\Rightarrow b=۲$

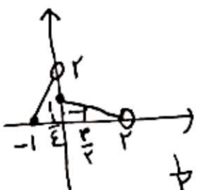
می‌دانیم بعد از صفر $\cos x$ نزولی است از آن جایی که در شکل صعودی است $a<0$ ۹

$\frac{۲+۴}{۲}=۳=|a|\Rightarrow c=-۱$

$\Rightarrow a=-۳$ $\quad f(\alpha)=-۳\cos ۲\alpha-۱=۰\Rightarrow \cos ۲\alpha=-\frac{۱}{۳}$

$f(x)=-۳\cos ۲x-۱$ $\quad \Rightarrow |\tan ۲\alpha|=۲\sqrt{۲}$

$y=f(x)$ $\quad T=۳$ $\quad f(-۳۴٫۵)=?$



$\forall x\in D_f \quad f(x)=f(x+T)$ ۱۰

$f(-۳۴٫۵)=f(-۳۴٫۵+۱۲T)=f(-۳۴٫۵+۳۶)=f(۱٫۵)$

$=\boxed{\frac{۱}{۴}}$

$\frac{\frac{۱}{۲}}{۲}=\frac{x}{۱}\Rightarrow x=\frac{۱}{۴}$