تکلیف شماره ۱۴ — مثلثات

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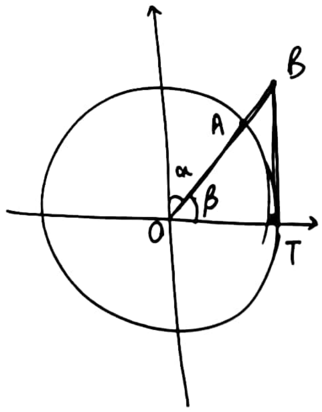

$OT = OA = 1$ شعاع دایره مثلثاتی

$\alpha + \beta = \frac{\pi}{2} \longrightarrow$ متمم اند

$\sin\alpha = \cos\beta = \frac{OT}{OB} = \frac{1}{OB} = \frac{1}{\underbrace{OA}_{1} + AB}$

$\sin\alpha = \frac{1}{1+AB} \longrightarrow 1 + AB = \frac{1}{\sin\alpha}$

$$AB = \frac{1}{\sin\alpha} - 1$$

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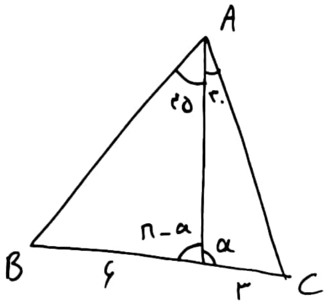

$\frac{3}{\underbrace{\sin 30}_{\frac{1}{2}}} = \frac{AC}{\sin\alpha} \longrightarrow AC = 6\sin\alpha$

$\frac{6}{\underbrace{\sin 45}_{\frac{\sqrt{2}}{2}}} = \frac{AB}{\underbrace{\sin(\pi-\alpha)}_{\sin\alpha}} \longrightarrow AB = 6\sqrt{2}\sin\alpha$

$\frac{AB}{AC} = \frac{6\sqrt{2}\sin\alpha}{6\sin\alpha} = \sqrt{2}$ جواب

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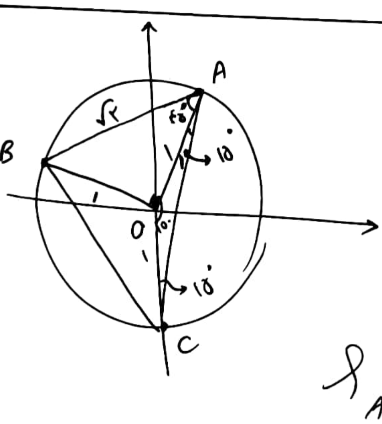

$OC = OA = OB = 1 \qquad \hat{A} = 60^\circ$

$A \left| \begin{matrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{matrix} \right. \qquad B \left| \begin{matrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{matrix} \right.$

الف) مساحت $\overset{\triangle}{ABC}$ ؟

$S_{\overset{\triangle}{ABC}} = \frac{1}{2} AB \times AC \times \sin\hat{A}$

$AB = \sqrt{2}$

$AC = \sqrt{\left(\frac{1}{2}-0\right)^2 + \left(\frac{\sqrt{3}}{2}-1\right)^2} = \sqrt{\frac{7-4\sqrt{3}}{4}} = \sqrt{\frac{(2-\sqrt{3})^2}{4}}$

$= \frac{2-\sqrt{3}}{2}$

$$S_{ABC}^{\triangle} = \frac{1}{2} \times \sqrt{2} \times \frac{2-\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2\sqrt{6} - \sqrt{12}}{8} = \frac{2\sqrt{6} - 2\sqrt{3}}{8}$$

ب) محیط $\triangle ABC$ ؟

$AB = \sqrt{2}$

$AC = \frac{2-\sqrt{2}}{2}$

$BC = \sqrt{\left(\frac{-\sqrt{2}}{2} - 0\right)^2 + \left(\frac{1}{2} - (-1)\right)^2} = \sqrt{\frac{2}{4} + \frac{9}{4}} = \sqrt{2}$

$$AB + AC + BC = \sqrt{2} + \sqrt{2} + \frac{2-\sqrt{2}}{2} = \frac{2\sqrt{2} + 2\sqrt{2} + 2 - \sqrt{2}}{2} = \frac{2 + 3\sqrt{2} + \sqrt{2}}{2}$$

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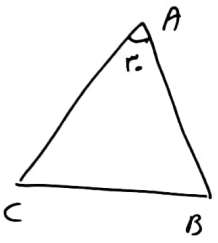

$AC = \log_{2}^{16} = 4 \log_{2}^{2}$

$AB = \log_{2}^{27} = 3 \log_{2}^{3}$

$$S_{ABC}^{\triangle} = \frac{1}{2} \times AC \times AB \times \underbrace{\sin \hat{A}}_{\frac{1}{2}} = \frac{1}{2} \times \frac{1}{2} \times \log_{2}^{16} \times \log_{2}^{27} = \frac{1}{2} \times \frac{1}{2} \times$$

$$\cancel{4 \log_{2}^{2}} \times \cancel{3 \log_{2}^{3}} = \frac{1}{4} \times 12 = \boxed{3}$$ جواب

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۵ -

if $x = \cos a \longrightarrow 2x^4 - x^2 - \sin^2 a \left(2\sin^2 a - 1\right) = 1$

$\longrightarrow 2\cos^4 x - \cos^2 x - \sin^2 x \left(2\sin^2 x - 1\right) = 1$

$$\underbrace{2\cos^4 x - 2\sin^4 x}_{2\left(\cos^4 x - \sin^4 x\right) = 2\left(\cos^2 x - \sin^2 x\right)\left(\cos^2 x + \sin^2 x\right) = 2\cos 2x} + \underbrace{\sin^2 x - \cos^2 x}_{-\cos 2x} = 1$$

$\longrightarrow 2\cos 2x - \cos 2x = 1 \longrightarrow \cos 2x = 1 \longrightarrow \boxed{\sin 2x = 0}$ جواب

$\cos^2 2x + \sin^2 2x = 1 \longrightarrow 1 + \sin^2 2x = 1$

$\sin^2 2x = 0 \longrightarrow \sin 2x = 0$

$y = -2\sin x \cos x + 2 \qquad [-\pi, \pi]$

$\sin x \cos x = \frac{1}{2}\sin 2x$

$y = -2 \times \frac{1}{2}\sin 2x + 2$

$y = -\sin 2x + 2 \qquad T = \frac{2\pi}{2} = \pi$

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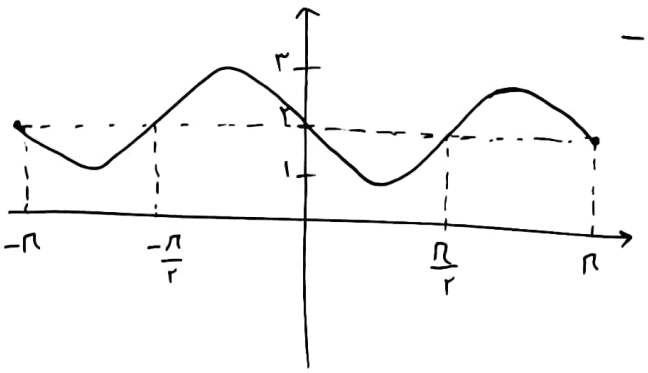
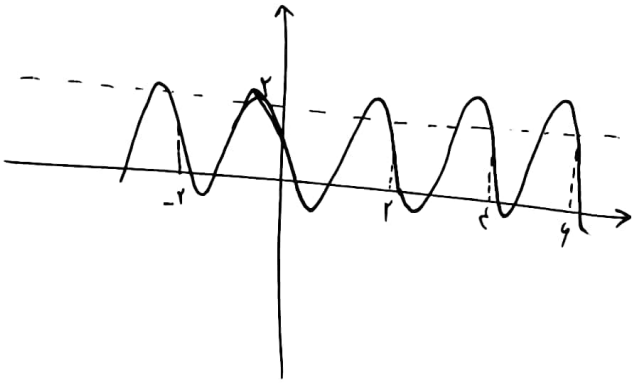


ب) $y = 2\cos\left(x + \frac{1}{2}\right)\pi + 1 \qquad [-\pi, 2\pi]$

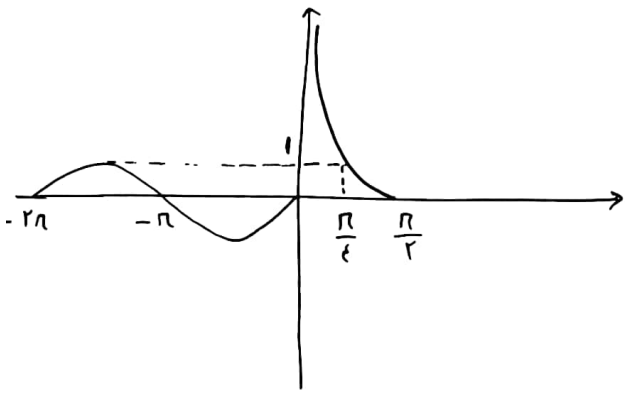
$y = 2\cos\left(x\pi + \frac{\pi}{2}\right) + 1 = -2\sin x\pi + 1$

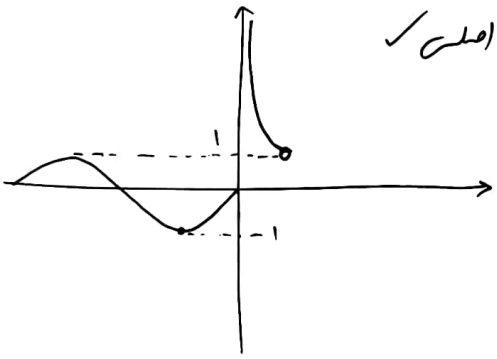
$T = \frac{2\pi}{\pi} = 2$



$$f(x) = \begin{cases} \cot x & 0 < x < \frac{\pi}{4} \\ \sin x & x \leq 0 \end{cases}$$

$R_f = [-1, +\infty)$ ۷ -





نمودار اصلی ✓

$f(x) = a + b\underbrace{\sin(cx)\cos(cx)}_{\frac{1}{2}\sin(2cx)}\underbrace{\cos(2cx)}_{\frac{1}{4}\sin(4cx)} = a + \frac{1}{4}b\sin(4cx)$ ۸ -

T طبق نمودار $= \frac{\pi}{2} - \frac{\pi}{10} = \frac{2\pi}{5} \longrightarrow T = \frac{2\pi}{|4c|} = \frac{2\pi}{5} \longrightarrow |4c| = 5$

$4c = \pm 5$

$c = \pm \frac{5}{4} \xleftarrow{c>0} c = \frac{5}{4}$

نکته

$f(x) = a + \frac{1}{4} b \sin(ax)$ $\quad \sin(\frac{5\pi}{2}) = 1$

if $x = \frac{\pi}{2} \longrightarrow f(x) = 4 \longrightarrow a + \frac{1}{4} b (\sin \frac{5\pi}{2}) = 4 \longrightarrow a + \frac{1}{4} b = 4$

if $x = \frac{3\pi}{10} \longrightarrow f(x) = 0 \longrightarrow a + \frac{1}{4} b \sin(\frac{3\pi}{2}) = 0 \longrightarrow a - \frac{1}{4} b = 0$

$\Rightarrow \begin{cases} a + \frac{1}{4} b = 4 \\ a - \frac{1}{4} b = 0 \end{cases}$

$2a = 4$

$a = 2$

$b = 8$

$\frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$ جواب

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$f(x) = a \cos bx + c$

چون نمودار cos است $\longrightarrow$ طبق نمودار $\longrightarrow T = \pi \longrightarrow T = \frac{2\pi}{|b|} = \pi \longrightarrow b = \pm 2$

علامت + و - b تفاوتی ایجاد نمی‌کند

$b = 2$

$f(x) = a \cos(2x) + c$

if $\cos 2x = 1 \longrightarrow f(x) = -4 \longrightarrow a + c = -4$

if $\cos 2x = -1 \longrightarrow f(x) = 2 \longrightarrow -a + c = 2$

$\Rightarrow 2c = -2$

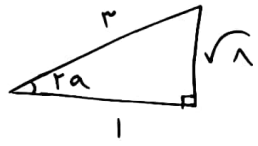
$c = -1$

$a = -3$

$f(x) = -3 \cos 2x - 1$

$f(a) = -3 \cos 2a - 1 = 0 \longrightarrow 3 \cos 2a = -1 \longrightarrow \cos 2a = \frac{-1}{3}$

$|\tan 2a| = \left| \frac{\sin 2a}{\cos 2a} \right| = \left| \frac{\frac{\sqrt{8}}{3}}{\frac{1}{3}} \right| = \sqrt{8}$ جواب



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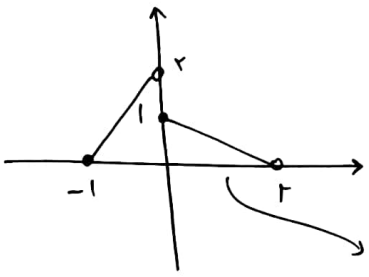

$T = 3$

$f(-3,5) = f(-3 + 1,5) = f(1,5) = f(\frac{3}{2})$

$\text{وسط} = \frac{-1}{2} \longrightarrow y = \frac{-1}{2} x + 1$

$y = \frac{-1}{2} (\frac{3}{2}) + 1 = \frac{1}{4}$ جواب