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نام و نام خانوادگی ......لیلی طباطبایی...... پاسخنامه تشریحی تکلیف شماره ..۱۴.. کلاس .سه‌شنبه — دوازدهم

۱

$m_{OA} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha \xrightarrow{\text{شیب خط مماس}} \frac{-1}{\tan\alpha} = -\cot\alpha = \frac{-\cos\alpha}{\sin\alpha}$

۱٫۵

$y - \sin\alpha = \frac{-\cos\alpha}{\sin}(\overset{1}{\cancel{x}} - \cos\alpha)$

$A(\cos\alpha, \sin\alpha)$, $B(1, y)$ $\Big\}$ $AB = \sqrt{(1-\cos\alpha)^2 + (y-\sin\alpha)^2} = \sqrt{(1-\cos\alpha)^2 + \left(-\frac{\cos\alpha}{\sin\alpha}(1-\cos\alpha)\right)^2} = |1-\cos\alpha|\sqrt{1+\frac{\cos^2\alpha}{\sin^2\alpha}}$

$= (1-\cos\alpha)\sqrt{\frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} \Rightarrow AB = \frac{2\sin^2\frac{\alpha}{2}}{\sin\alpha} = \boxed{\frac{1-\sqrt{1-\sin^2\alpha}}{\sin\alpha}}$

۲

$\frac{AB}{\sin \cancel{ADB}\,\theta} = \frac{BD}{\sin 45}$ و $\frac{AC}{\sin\cancel{ADC}\,\theta} = \frac{DC}{\sin 30} \Rightarrow AB = \frac{4\sin\theta}{\sin 45}, AC = \frac{3\sin\theta}{\sin 30}$

$\frac{AB}{AC} = \frac{\frac{4\sin\theta}{\sin 45}}{\frac{3\sin\theta}{\sin 30}} = 2 \times \frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \boxed{\sqrt{2}}$ ✓

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الف)

$x^2+y^2=1 \Rightarrow x_A = \frac{1}{2}, y_A = \sqrt{1-(\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$, $C = (0, -1)$

$y_B = \frac{1}{2} \Rightarrow x_B = -\sqrt{1-(\frac{1}{2})^2} = -\frac{\sqrt{3}}{2}$

مساحت مثلث: $\frac{1}{2}|x_A(y_B-y_C) + x_B(y_C-y_A) + x_C(y_A-y_B)|$

$= \frac{1}{2}\left|\frac{3}{4}+\frac{3}{4}+\frac{\sqrt{3}}{2}\right| = \frac{1}{2} \times \left|\frac{3}{2}+\frac{\sqrt{3}}{2}\right| = \boxed{\frac{3+\sqrt{3}}{4}}$ ✓

۲

$AB = \sqrt{(\frac{1}{2}+\frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2}-\frac{1}{2})^2} = \frac{1}{2}\sqrt{(1+\sqrt{3})^2+(\sqrt{3}-1)^2} = \frac{1}{2}\sqrt{8} = \boxed{\sqrt{2}}$ $BC = \sqrt{(-\frac{\sqrt{3}}{2}-0)^2 + (\frac{1}{2}-(-1))^2} = \boxed{\sqrt{3}}$ ✓

$CA = \sqrt{(\frac{1}{2}-0)^2 + (\frac{\sqrt{3}}{2}-(-1))^2} = \frac{1}{2}\sqrt{1+7+4\sqrt{3}} = \boxed{\sqrt{2+\sqrt{3}}} \Rightarrow \boxed{P = \sqrt{2}+\sqrt{3}+\sqrt{2+\sqrt{3}}}$ ✓

۴

$AB = 3\log_2^3 = \frac{3}{\log_3^2}$

$AC = 4\log_3^2$

$S_{ABC} = \frac{1}{2}AB \cdot AC \sin A = \frac{1}{2} \times \frac{3}{\cancel{\log_3^2}} \times 4\cancel{\log_3^2} = \boxed{6}$

$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin\frac{\pi}{6} = \frac{1}{4} \times \log_2^{27} \times \log_3^{16}$

$\frac{1}{\cancel{4}} \times 3\log_2^3 \times \cancel{4}\log_3^2 = 3\log_{\cancel{2}}^{\cancel{3}} \times \log_{\cancel{3}}^{\cancel{2}} = \underline{3}$

۱٫۷۵

$2\cos^4\alpha - \cos^2\alpha - \sin^2(\alpha)(2\sin^2\alpha - 1) = 1 \xrightarrow{\sin^2\alpha = 1-\cos^2\alpha} 2\cos^4\alpha - \cos^2\alpha - (1-\cos^2\alpha)(1-2\cos^2\alpha) = 1$

$\Rightarrow \cancel{2\cos^4\alpha} - \cos^2\alpha - (\cancel{2\cos^4\alpha} - 3\cos^2\alpha + 1) \Rightarrow 2\cos^2\alpha - 1 = 1 \Rightarrow \cos^2\alpha = 1 \Rightarrow \cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow \sin^2\alpha = 0 \Rightarrow \sin\alpha = 0 \Rightarrow \sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \times 0 \times 1 = \boxed{0}$ ✓

۲

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۶

الف)

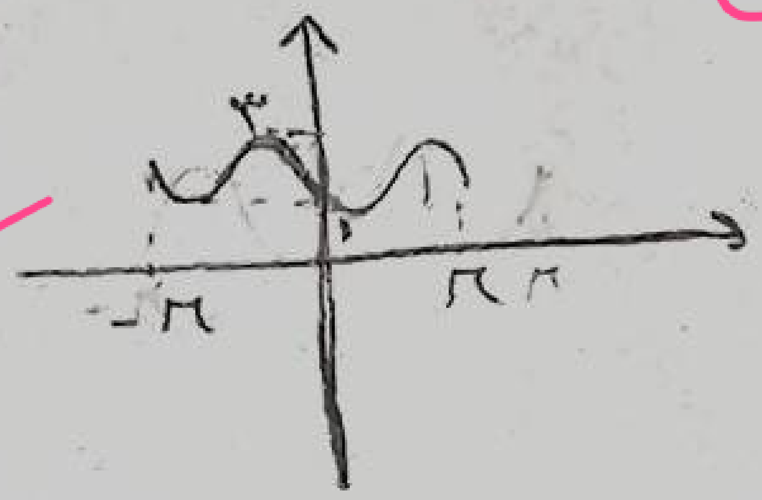
$y=-۲\sin\alpha\cos\alpha+۲=-\sin ۲x+۲$

$\frac{۲\pi}{۲}=\pi=T$

$\min=-|-۱|+۲=۱$

$\max=|-۱|+۲=۳$

✓ ۲



$y=۲\cos(\pi x+\frac{\pi}{۲})+۱$

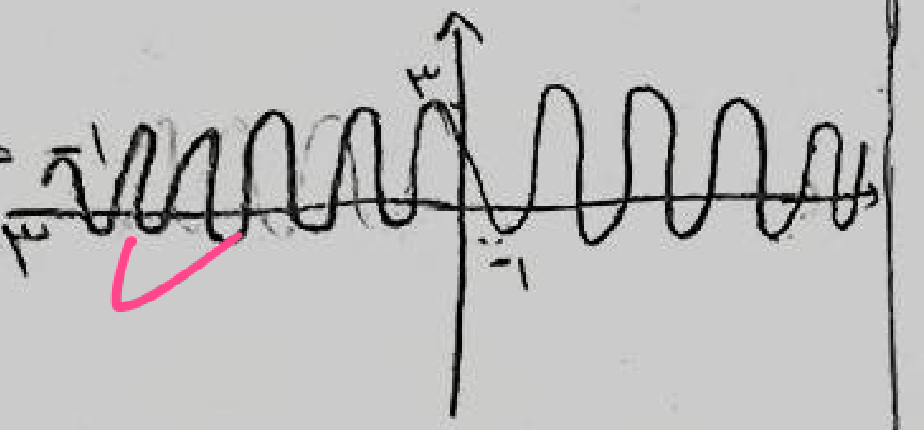
$y=-۲\sin\pi x+۱$

$\frac{۲\pi}{\pi}=۲=T$

$\min=-|-۲|+۱=-۱$

$\max=|-۲|+۱=۳$

✓



۷

$\cot ۰=+\infty$

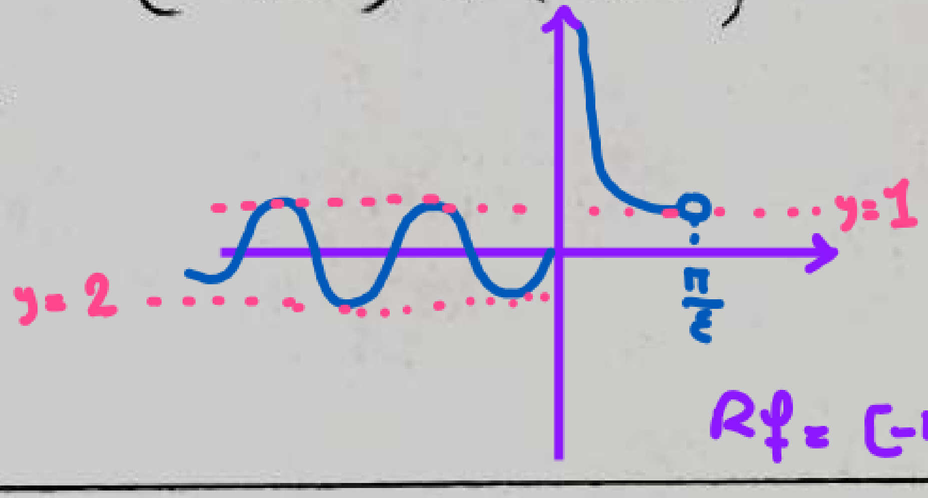
$\cot\frac{\pi}{۴}=۱$

$\Big\}\ R_f=(۱,+\infty)$

$\sin x \rightarrow x\leq ۰ \Rightarrow R_f=[-۱,۰]$

$\Big\}\ \cup=[-۱,۰]\cup(۱,+\infty)$

۱,۵



$Rf=[-1,+\infty)$

۸

$f(x)=a+b\underbrace{\sin(cx)\cos(cx)}_{\frac{۱}{۲}\sin(۲cx)}\cos(۲cx)=a+\frac{b}{۴}\sin(۴cx)$

$\underbrace{\frac{۱}{۲}\sin(۲cx)\cos(۲cx)}_{\frac{۱}{۴}\sin(۴cx)}$

$\frac{bc}{a}=\frac{\frac{۵}{۴}\times ۸}{۲}=\boxed{۵}$ ✓ ۲

نمودار با صعود شروع می‌شود

a و b هم علامت

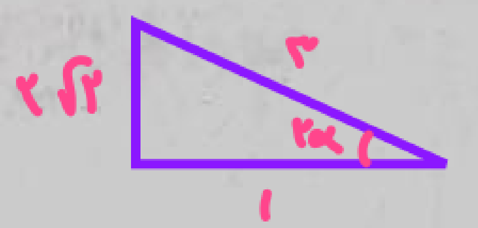
$T=\frac{\pi}{۲}-\frac{\pi}{۱۰}=\frac{۲\pi}{۵}\Rightarrow\frac{۲\pi}{\frac{۲\pi}{۵}}=۴c\Rightarrow c=\frac{۵}{۴}$ و $\begin{cases}-|\frac{b}{۴}|+a=۰\\ |\frac{b}{۴}|+a=۴\end{cases}$ $۲a=۴\Rightarrow a=۲\ \Big\}\ b=۸$

۹

$a<۰\rightarrow$ با صعود شروع شدن

$f(۲\alpha)<۰\rightarrow -۳\cos ۲\alpha-۱<۰\rightarrow\cos ۲\alpha=-\frac{۱}{۳}$

$f(x)=a\cos bx+c$



$\rightarrow |\tan ۲\alpha|=۲\sqrt{۲}$

۱,۵

$f(x)=-۳\cos ۲x-۱$

$\frac{۲\pi}{b}=\pi\Rightarrow b=۲$

$\begin{cases}|a|+c=۲\\ -|a|+c=-۴\end{cases}$ $۲c=-۲\Rightarrow c=-۱\Rightarrow |a|=۳\Rightarrow a=-۳$

۱۰

$f(x)=y\quad T=۳$

$f(x+T)=f(x)$

$f(x)\begin{cases}-\frac{۱}{۲}x+۱ & ;\ ۰\leq x\leq ۲\\ ۲x+۲ & ;\ -۱\leq x<۰\end{cases}$

$\rightarrow f(۱,۵)=-\frac{۱}{۲}(\frac{۳}{۲})+۱=\frac{۱}{۴}$

۱,۷۵

$-۳۴,۵+۳(۱۲)=x=۱,۵\Rightarrow f(۱,۵)=f(-۳۴,۵)=\boxed{۱,۷۵}$

$\underset{[۰,۲)}{f(x)}\ \frac{۱}{۲}x+۱=\frac{۱}{۲}\times ۱,۵+۱$

1

OT و OA شعاع‌های دایره هستند و طول آنها یک است.

$$BT = \tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$$

$$OB^2 = OT^2 + BT^2 \rightarrow OB^2 = 1^2 + \cot^2\alpha \rightarrow OB^2 = 1 + \cot^2\alpha$$

$$OB^2 = \frac{1}{\sin^2\alpha} \rightarrow OB = \frac{1}{|\sin\alpha|} \xrightarrow{\alpha \text{ حاده است}} OB = \frac{1}{\sin\alpha}$$

$$AB + 1 = \frac{1}{\sin\alpha} \rightarrow AB = \frac{1}{\sin\alpha} - 1 = \frac{1 - \sin\alpha}{\sin\alpha}$$