سارا یاسکری

(۱)

$$\cos\beta_1 = \sin\alpha = \frac{OT}{OB} = \frac{1}{1+AB} \Rightarrow \sin\alpha + \sin\alpha(AB) = 1$$

$$\Rightarrow AB = \frac{1-\sin\alpha}{\sin\alpha}$$

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(۲)

$$\frac{3}{\frac{1}{2}} = \frac{AC}{\sin\alpha} \qquad \frac{6}{1} = \frac{AB}{\sin(90-\alpha)} \qquad \frac{AB}{AC} = \frac{6\cos\alpha}{6\sin\alpha} = \cot\alpha$$

$$\sin(90-\alpha) = \cos\alpha$$

$$\Rightarrow AC = 6\sin\alpha \qquad \Rightarrow AB = 6\cos\alpha$$

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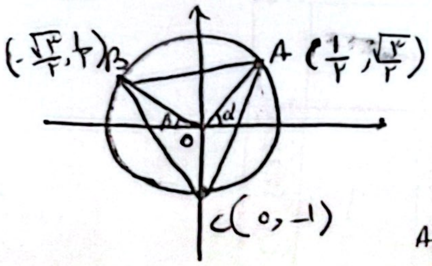
(۳)



$$\cos\alpha = \frac{1}{2} \xRightarrow{\sin^2\alpha+\cos^2\alpha=1} \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60^\circ$$

$$\sin\beta = \frac{1}{2} \xRightarrow{\sin^2\beta+\cos^2\beta=1} \cos\alpha = -\frac{\sqrt{3}}{2} \Rightarrow \beta = 150^\circ$$

خط AC: $ax+b \Rightarrow \left.\begin{array}{l} \frac{a}{2}+b = \frac{\sqrt{3}}{2} \\ b=-1 \end{array}\right\} \Rightarrow a = \sqrt{3}+2$

$$\downarrow$$

$$(\sqrt{3}+2)x - 1$$

فاصله B از خط AC $= \dfrac{(\sqrt{3}+2)(-\frac{\sqrt{3}}{2}) - \frac{1}{2} - 1}{\sqrt{3+4+4\sqrt{3}+1}} = \dfrac{-\frac{3}{2} - \sqrt{3} - \frac{1}{2} - 1}{2\sqrt{2+\sqrt{3}}} = \dfrac{-2-\sqrt{3}}{2\sqrt{2+\sqrt{3}}} \times \sqrt{2+\sqrt{3}} \times \frac{1}{2}$

$$8+4\sqrt{3}$$

$$= \frac{-2-\sqrt{3}}{4}$$

(الف)

فاصله C,A $= \sqrt{\frac{1}{4} + \left(\frac{\sqrt{3}+2}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4} + 1 + \sqrt{3}} = \sqrt{2+\sqrt{3}}$

فاصله BA $= \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}\right)^2} = \sqrt{1+\frac{9}{4}} = \frac{\sqrt{10}}{2}$

فاصله CB $= \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$

محیط $= \sqrt{2+\sqrt{3}} + \frac{\sqrt{10}}{2} + \sqrt{3}$

(ب)

(۴)

$$S_\Delta = \underbrace{\log_2^3 \times \log_2^2 \times \frac{1}{2} \times \frac{1}{2}}_{3\log_2^2 \times \frac{4}{\log_2^3}} = 12$$

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(۵)

$$f(\cos\alpha) = 2\cos^4\alpha - \cos^2\alpha - \underbrace{\sin^2\alpha(2\sin^2\alpha - 1)}_{-2\sin^4\alpha + \sin^2\alpha} = 1$$

$$\Rightarrow \underbrace{2\cos^4\alpha - 2\sin^4\alpha}_{2(\cos^2\alpha - \sin^2\alpha)} - \cos^2\alpha + \sin^2\alpha = \cos^2\alpha - \sin^2\alpha = 1 \Rightarrow \cos 2\alpha = 1$$

$$\Rightarrow \sin^2\alpha = \sqrt{1-1} = 0$$

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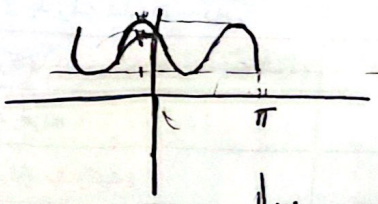
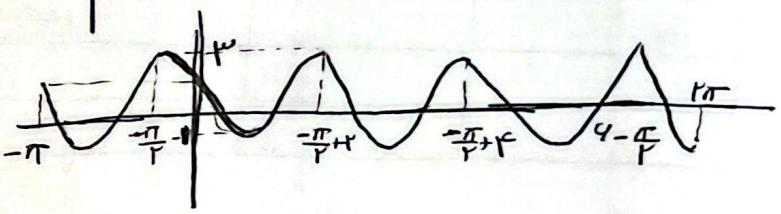
(۶)

الف) $-\sin^2 x + 2 \quad T = \frac{2\pi}{2} = \pi$



ب) $2\cos\left(\pi x + \frac{\pi}{2}\right) + 1 \quad T = \frac{2\pi}{\pi} = 2$



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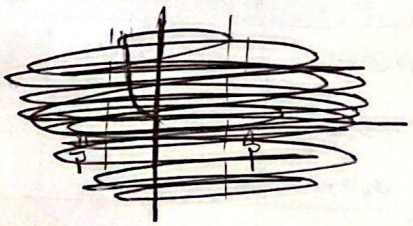
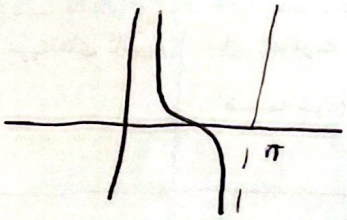
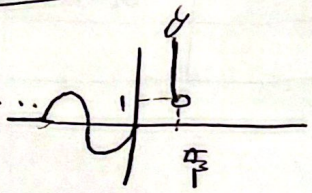
(۷)

$\cot x$





$\cot x \quad \alpha < x < \frac{\pi}{2} \Rightarrow$



$R_{\cot x} = (+\infty, -1]$

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(۸)

$$f(x)\ a + \frac{b}{2}\sin(2cx)\cos 2cx = a + \frac{b}{4}\sin 4cx$$

$$T = \frac{\pi}{2} - \frac{\pi}{10} = \frac{2\pi}{5} = \frac{2\pi c}{|4c|} \Rightarrow c = \frac{5}{4}$$

$$\frac{bc}{a} = \frac{4 \times \frac{5}{4}}{2} = 2/5$$

$$\max = a + \left|\frac{b}{4}\right| = a + \frac{b}{4} = 5$$

$$\Rightarrow a - \frac{b}{4} = 0$$

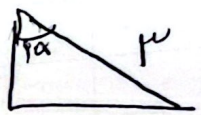
$$\left.\begin{matrix} a = 2 \\ b = 4 \end{matrix}\right\}$$

$T=\pi \Rightarrow \frac{2\pi}{b}=\pi \Rightarrow b=2$

$$\left.\begin{array}{l} c+|a|=c-a=2 \\ c-|a|=c+a=-4 \end{array}\right\} \Rightarrow \begin{array}{l} c=-1 \\ a=-3 \end{array}$$

(4)

$-2\cos 2x-1=0 \Leftrightarrow \cos 2x=-\frac{1}{2} \Rightarrow \cos^2 2x=\frac{1}{4}$



$\Rightarrow \tan 2x=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$

$f(-2\pi,a)=f\left((+2)(-11)+1,a\right)=f(1,a)=1,a\times-\frac{1}{2}+1=$ (10) (4,2a)

$-\frac{1}{2}a+1$ biggest