۱۸,۲۵

سارا پاسترکی

① ۲

$$\cos\beta_1 = \sin\alpha = \frac{OT}{OB} = \frac{1}{1+AB} \Rightarrow \sin\alpha + \sin\alpha(AB) = 1$$

$$\Rightarrow AB = \frac{1-\sin\alpha}{\sin\alpha}$$

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② ۱,۲۵

$$\frac{3}{\frac{1}{2}} = \frac{AC}{\sin\alpha} \qquad \frac{6}{1} = \frac{AB}{\underbrace{\sin(90-\alpha)}_{=\cos\alpha}} \qquad \frac{AB}{AC} = \frac{6\cos\alpha}{6\sin\alpha} = \cot\alpha$$

$$\Rightarrow AC = 6\sin\alpha \qquad \Rightarrow AB = 6\cos\alpha$$

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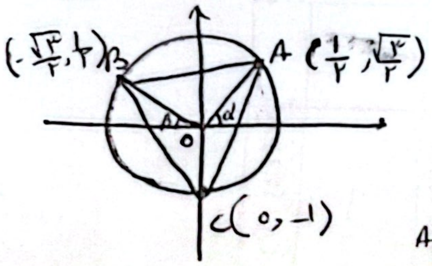
③ ۱,۵



$$\cos\alpha = \frac12 \xRightarrow{\sin^2\alpha+\cos^2\alpha=1} \sin\alpha = \frac{\sqrt3}{2} \Rightarrow \alpha = 60^\circ$$

$$\sin\beta = \frac12 \xRightarrow{\sin^2\alpha+\cos^2\alpha=1} \cos\alpha = -\frac{\sqrt3}{2} \Rightarrow \beta = 150^\circ$$

خط AC: $ax+b \Rightarrow \left.\begin{array}{l}\frac{a}{2}+b = \frac{\sqrt3}{2}\\ b=-1\end{array}\right\} \Rightarrow a = \sqrt3+2$

$\downarrow$ $(\sqrt3+2)x-1$

فاصله B از خط AC $= \dfrac{(\sqrt3+2)(-\frac{\sqrt3}{2}) - \frac12 - 1}{\sqrt{\underbrace{3+4+4\sqrt3+1}_{8+4\sqrt3}}} = \dfrac{-\frac32 - \sqrt3 - \frac12 - 1}{2\sqrt{2+\sqrt3}} = \dfrac{-2-\sqrt3}{2\sqrt{2+\sqrt3}} \times \sqrt{2+\sqrt3} \times \frac12 = \dfrac{-2-\sqrt3}{4}$

الف

فاصله C, A $= \sqrt{\frac14 + \left(\frac{\sqrt3+2}{2}\right)^2} = \sqrt{\frac14 + \frac34 + 1 + \sqrt3} = \sqrt{2+\sqrt3}$

فاصله BA $= \sqrt{\left(\frac{1+\sqrt3}{2}\right)^2 + \left(\frac{\sqrt3-1}{2}\right)^2} = \sqrt{1+\frac94} = \frac{\sqrt{10}}{2}$

فاصله CB $= \sqrt{\frac34 + \frac94} = \sqrt3$

محیط $= \sqrt{2+\sqrt3} + \frac{\sqrt{10}}{2} + \sqrt3$

ب

۴

$S_\Delta = \underbrace{۳۲\log_۲^۳ \times ۴\log_۳^۲}_{} \times \frac{۱}{۲} \times \frac{۱}{۴} = ۳$

$۳۲\log_۲^۳ \times \frac{۴}{\log_۲^۳} = ۱۲$

۲

۵

$f(\cos\alpha) = ۲\cos^۴\alpha - \cos^۲\alpha - \underbrace{\sin^۲\alpha\,(۲\sin^۲\alpha - ۱)}_{-۲\sin^۴\alpha + \sin^۲\alpha} = ۱$

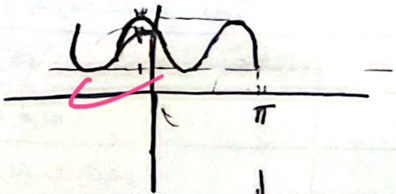
۲

$\Rightarrow \underbrace{۲\cos^۴\alpha - ۲\sin^۴\alpha}_{۲(\cos^۲\alpha - \sin^۲\alpha)} - \cos^۲\alpha + \sin^۲\alpha = \cos^۲\alpha - \sin^۲\alpha = ۱ \Rightarrow \cos ۲\alpha = ۱$

$\Rightarrow \sin ۲\alpha = \sqrt{۱-۱} = ۰$

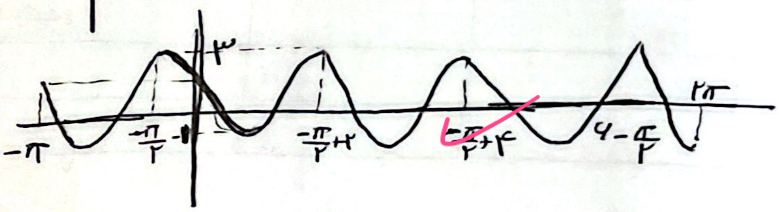
۶

الف) $-\sin ۲x + ۲ \quad T = \frac{۲\pi}{۲} = \pi$



۲

ب) $۲\cos(\pi x + \frac{\pi}{۴}) + ۱ \quad T = \frac{۲\pi}{\pi} = ۲$



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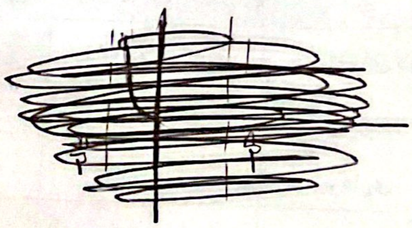
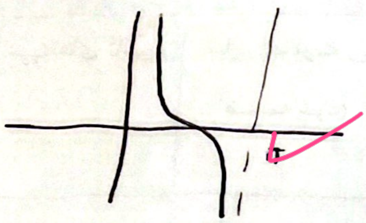
Cotx





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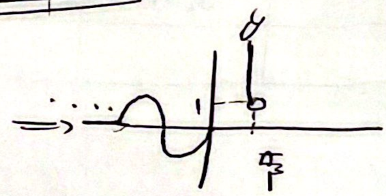
Cotx $0 < x < \frac{\pi}{۴}$ $\Rightarrow$



$R_f = (+\infty, -۱]$

$[-۱, +\infty)$

۸

۱/۷۵

$f(x)\ a + \frac{b}{۲}\sin(۲cx)\cos ۲cx = a + \frac{b}{۴}\sin ۴cx$ — چون عدد در جایی عضو مثبت است

پس b,c مثبت است —

$T = \frac{\pi}{۲} - \frac{\pi}{۱۰} = \frac{۲\pi}{۵} = \frac{۲\pi}{|۴c|} \xRightarrow{c>0} c = \frac{۵}{۴}$

$\max = a + \left|\frac{b}{۴}\right| = a + \frac{b}{۴} = ۴$

$\Rightarrow$

$a - \frac{b}{۴} = ۰$

$a = ۲$, $b = ۸$

$\frac{bc}{a} = \frac{\cancel{۸} \times \frac{۵}{۴}}{۲} = ۵$

$\frac{b}{۴} = ۲ \rightarrow b = ۸$

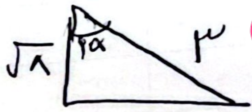
$T=\pi \Rightarrow \frac{2\pi}{b} = \pi \Rightarrow b=2$

$$\left.\begin{array}{l} c+|a| = c-a = 2 \\ c-|a| = c+a = -4 \end{array}\right\} \Rightarrow \begin{array}{l} c=-1 \\ a=-3 \end{array}$$

۴

۱/۷۵

$-3\cos 2x - 1 = 0 \Rightarrow \cos 2x = -\frac{1}{3} \Rightarrow \cos 2\alpha = -\frac{1}{3}$



$\Rightarrow |\tan 2\alpha| = \frac{1}{\sqrt{8}} = \frac{\sqrt{8}}{8}$

$\cos 2\alpha = -\frac{1}{3}$

$|\tan 2\alpha| = 2\sqrt{2}$

۵

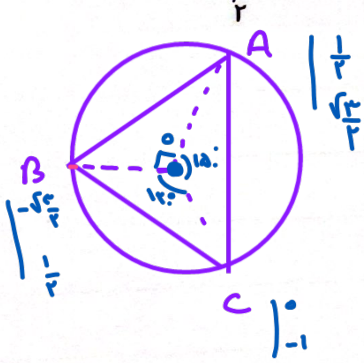
$f(-31/5) = f((+3)(-11)+1/5) = f(1/5) = 1/5x - \frac{1}{2} + 1 = 0/25$

درسته

۲

$-\frac{1}{2}a+1$

سوال ۳



$S_{AOC} = 1\times 1\times \frac{1}{2}\times \frac{1}{2} = \frac{1}{4}$

$S_{OAB} = 1\times 1\times \frac{1}{2} = \frac{1}{2}$

$S_{OBC} = 1\times 1\times \frac{1}{2}\times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$S_{ABC} = \frac{1}{2} + \frac{1}{4} + \frac{\sqrt{3}}{4} = \frac{3+\sqrt{3}}{4}$

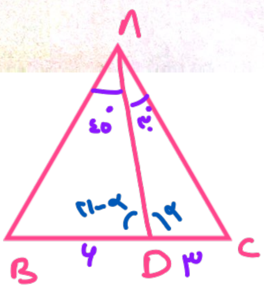
$BC^2 = 1^2 + 1^2 - 1\times 1\times 2\times -\frac{1}{2} = 1+1+1 = 3 \rightarrow BC = \sqrt{3}$

$AC^2 = 1^2 + 1^2 - 1\times 1\times 2\times -\frac{\sqrt{3}}{2} = 1+1+\sqrt{3} \rightarrow AC = \sqrt{2+\sqrt{3}}$

$AB^2 = 1^2 + 1^2 \rightarrow AB = \sqrt{2}$

$P_{ABC} = \sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$

۴-



$ADC \rightarrow \frac{\sin\hat{\alpha}}{AC} = \frac{\sin 30}{DC} \rightarrow AC = 6\sin\hat{\alpha}$

$ABD \rightarrow \frac{\sin(\pi-\alpha)}{AB} = \frac{\sin 45}{4} \rightarrow AB = 4\sqrt{2}\sin\hat{\alpha}$

$\frac{AB}{AC} = \frac{4\sqrt{2}\sin\alpha}{6\sin\alpha} = \sqrt{2}$