پارمیس بنی زاده، دوازدهم دفتر B — تکلیف شماره ۱۴

۱) $A = (\cos(۹۰-\alpha) , \sin(۹۰-\alpha)) = (\sin\alpha , \cos\alpha)$

$B = (۱ , \tan(۹۰-\alpha)) = (۱ , \cot\alpha)$

$\Rightarrow AB = \sqrt{(۱-\sin\alpha)^۲ + (\cot\alpha - \cos\alpha)^۲}$

$\cot\alpha - \cos\alpha = \cos\alpha\left(\frac{1}{\sin\alpha} - ۱\right) = \cos\alpha\left(\frac{۱-\sin\alpha}{\sin\alpha}\right)$

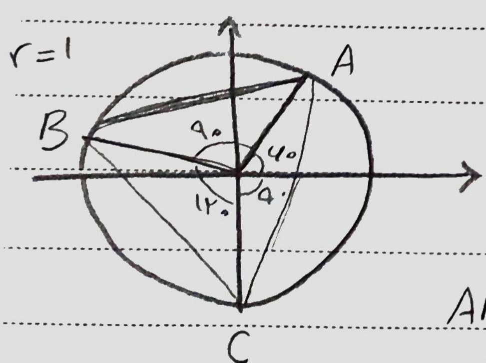
$AB = |۱-\sin\alpha|\sqrt{۱ + \frac{\cos^۲\alpha}{\sin^۲\alpha}} = (۱-\sin\alpha) \times \frac{1}{\sin\alpha}$

۲) $\frac{AB}{\sin(۱۸۰-\alpha)} = \frac{BD}{\sin ۴۵} = ۴\sqrt{۲} \Rightarrow \frac{AB}{\sin\alpha} = ۴\sqrt{۲}$

$\frac{AC}{\sin\alpha} = \frac{DC}{\sin ۳۰} = ۴ \longrightarrow \frac{AC}{\sin\alpha} = ۴$

$\frac{AB}{\sin\alpha} \div \frac{AC}{\sin\alpha} = \frac{۴\sqrt{۲}}{۴} = \sqrt{۲} \Rightarrow \frac{AB}{AC} = \sqrt{۲}$

۳) $A = \left(\frac{۱}{۲} , \frac{\sqrt{۳}}{۲}\right) \to A = ۶۰^\circ \qquad B = \left(\frac{-\sqrt{۳}}{۲} , \frac{۱}{۲}\right) \to B = ۱۵۰$



الف)

$S_{ABO} = \frac{۱}{۲} \times ۱ \times ۱ = \frac{۱}{۲}$

$S_{BOC} = \frac{۱}{۲} \times ۱ \times ۱ \times \frac{\sqrt{۳}}{۲} = \frac{\sqrt{۳}}{۴}$

$S_{AOC} = \frac{۱}{۲} \times ۱ \times ۱ \times \frac{۱}{۲} = \frac{۱}{۴}$

$S_{ABC} = \frac{۳+\sqrt{۳}}{۴}$

ب)

$AB = \sqrt{۱+۱-۰} = \sqrt{۲}$

$BC = \sqrt{۱+۱-۲(-\frac{۱}{۲})} = \sqrt{۳}$

$AC = \sqrt{۱+۱-۲(\frac{-\sqrt{۳}}{۲})} = \sqrt{۲+\sqrt{۳}}$

محیط مثلث $= \sqrt{۲} + \sqrt{۳} + \sqrt{۲+\sqrt{۳}}$

۴) $S_{\overset{\Delta}{ABC}} = \frac{1}{2} \times \log_2^{27} \times \log_3^{16} \times \frac{1}{2}$

$\Rightarrow S = \frac{1}{2} \times 3\log_2^{3} \times 4\log_3^{2} \times \frac{1}{2} = 3\log_2^{3} \times \frac{1}{\log_2^{3}} = 3$

۵) $x^2(2x^2-1) - \sin^2\alpha(2\sin^2\alpha - 1)$

به جای x، $(\cos\alpha)$ قرار میدهیم $\rightarrow 2\cos^4\alpha - \cos^2\alpha - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$\Rightarrow \cos^2\alpha(2\cos^2\alpha - 1) - \sin^2\alpha(2\sin^2\alpha - 1) = 1$

$\cos^2\alpha(\cos^2\alpha - \sin^2\alpha) - \sin^2\alpha(\sin^2\alpha - \cos^2\alpha) = 1$

$\cos^2\alpha(\cos^2\alpha - \sin^2\alpha) + \sin^2\alpha(\cos^2\alpha - \sin^2\alpha) = 1$

$(\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha) = 1 \Rightarrow \cos^2\alpha - \sin^2\alpha = 1$

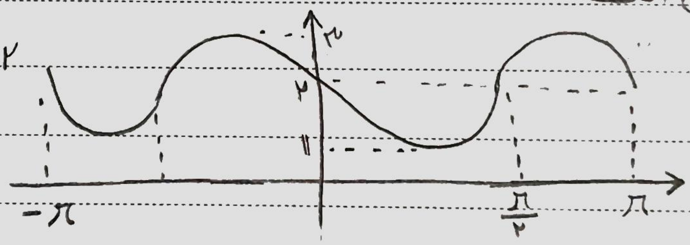
$\cos 2\alpha = 1 \qquad \sin 2\alpha = 0$

۶) الف

$y = -2\sin x\cos x + 2$

$y = -\sin 2x + 2$

$T = \frac{2\pi}{2} = \pi$

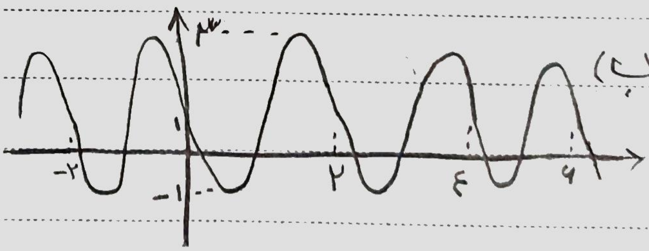


ب)

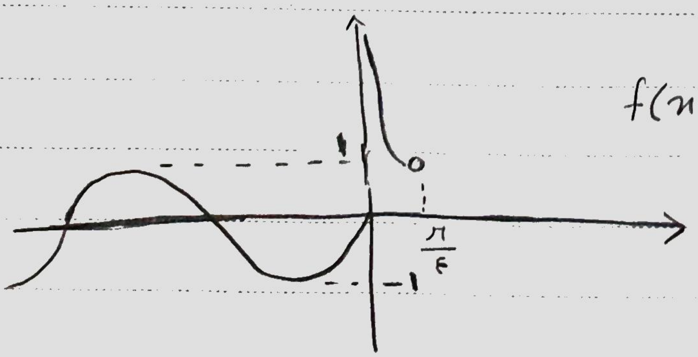
$y = 2\cos(\pi x + \frac{\pi}{2}) + 1$

$y = 2(-\sin \pi x) + 1$

$T = \frac{2\pi}{\pi} = 2$



۷) $f(x) = \begin{cases} \cot x \rightarrow (1, \infty) \\ \sin x \rightarrow [-1, 1] \end{cases}$



$R_f = [-1, +\infty)$

$$f(x) = a + b\,\sin(cx)\cos(cx)\cos(2cx) \quad (8$$

$$f(x) = a + b\left(\tfrac{1}{2}\sin(2cx)\cos(2cx)\right)$$

$$f(x) = a + b\left(\tfrac{1}{4}\sin(4cx)\right)$$

$$f(x) = a + \frac{b}{4}\left(\sin(4cx)\right)$$

$$T = \frac{2\pi}{|4c|} = \frac{2\pi}{5} \Rightarrow |4c| = 5 \xrightarrow{c>0} c = \frac{5}{4}$$

$$\left.\begin{aligned} \max &= a + \frac{b}{4} = 4 \\ \min &= a - \frac{b}{4} = 0 \end{aligned}\right\} \quad a = 2 \quad b = 8$$

$$\frac{bc}{a} = \frac{8 \times \frac{5}{4}}{2} = 5$$

(9 علامتش چون منفیست $\longrightarrow$ $T = \frac{2\pi}{|b|} = \pi \Rightarrow |b| = 2$

$$\left.\begin{aligned} \max &= -a + c = 2 \\ \min &= +a + c = -4 \end{aligned}\right\} \Rightarrow c = (-1) \quad a = (-3)$$

$$f(x) = -3\cos 2x - 1$$

$$f(\alpha) \rightarrow -3\cos 2\alpha - 1 = 0 \Rightarrow \cos 2\alpha = \frac{-1}{3}$$

$$1 + \tan^2 2\alpha = \frac{1}{\cos^2 2\alpha} \Rightarrow \tan^2 2\alpha = 9 - 1 = 8$$

$$|\tan\alpha| = \sqrt{8} = 2\sqrt{2}$$

(10

$$f(-34.5) = f(1.5)$$

معادله خط زرد $= \frac{-1}{2}x + 1$

$$f(1.5) = \frac{-1}{2}\left(\frac{3}{2}\right) + 1 = \frac{1}{4}$$