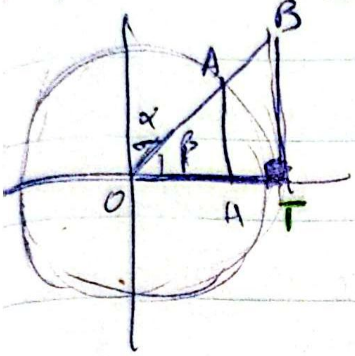
به نام خدا — نیلو حیدری — شیت (۱٤)

سوال ۱

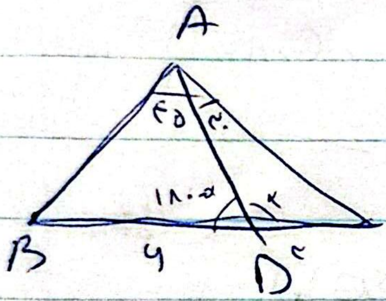


$\alpha + \beta = 90 \rightarrow \sin\beta = \cos\alpha$

$\frac{AH}{BT} = \frac{OH}{OT} = \frac{OA}{OB} \rightarrow \frac{\sin\beta}{1} = \frac{1}{1+AB}$

$\sin\beta + AB\sin\beta = 1 \rightarrow AB = \frac{1-\sin\beta}{\sin\alpha}$

سوال ۲



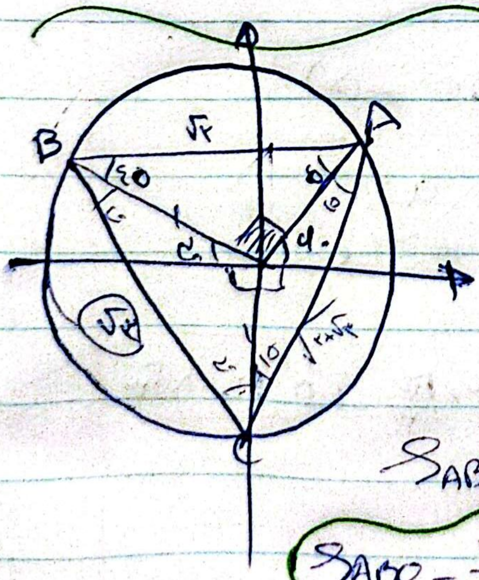
$\frac{AC}{\sin\alpha} = \frac{3}{\sin 30} \rightarrow \frac{AC}{\sin C} = 6$

$\frac{1}{2}$

$\frac{AB}{\sin(180-\alpha)} = \frac{6}{\frac{\sqrt{2}}{2}} \rightarrow \frac{12\sqrt{2}}{2} \rightarrow 6\sqrt{2}$

$\frac{AB}{AC} \rightarrow \frac{\frac{AB}{\sin\alpha}}{\frac{AC}{\sin\alpha}} \rightarrow \frac{6\sqrt{2}}{6} = \boxed{\sqrt{2}}$

سوال ۳



$AB = \sqrt{1+1} \rightarrow \sqrt{2}$

$BC = \sqrt{1+1 - 2\times -\frac{1}{2} \times 1} \rightarrow \sqrt{3}$

$AC = \sqrt{1+1 - 2\times \frac{\sqrt{3}}{2} \times 1} \rightarrow \sqrt{2+\sqrt{3}}$

$S_{ABC} = \frac{1}{2} ab\sin C \rightarrow \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \sqrt{3} \times \sqrt{2+\sqrt{3}}$

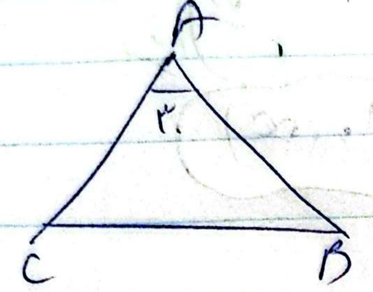
$S_{ABC} = \frac{\sqrt{6} \times \sqrt{2+\sqrt{3}}}{4}$

الف

$\sqrt{2} + \sqrt{3} + \sqrt{2+\sqrt{3}}$

ب

سوال ۴



$\frac{1}{2} AB \cdot AC \sin\alpha \Rightarrow \frac{1}{2}\log_{2} 27 \cdot \log_{3} 16 \times \sin 30$

$\frac{1}{2}\times\frac{1}{2}\times 3\log_{2} 3 \times 4\log_{3} 2 = 3$

سوال ۵

$x^2(x^2-1) - \sin^2(\alpha)(2\sin^2\alpha - 1) \Rightarrow x^2(x^2-1) + \sin^2\alpha\cos 2\alpha$

$-\cos 2\alpha$

$x \to \cos\alpha$

$\cos^2\alpha(2\cos^2\alpha - 1) + \sin^2\alpha\cos 2\alpha = \cos^2\alpha\cos 2\alpha + \sin^2\alpha\cos 2\alpha$

$1+\cos 2\alpha$

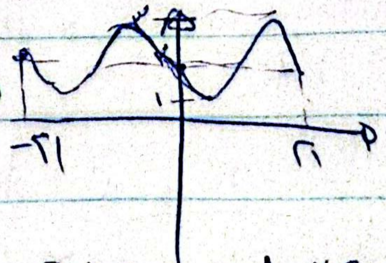
$= \cos 2\alpha(\sin^2\alpha + \cos^2\alpha) = \cos 2\alpha = 1$

$1 - \cos^2\alpha - \sin^2\alpha \to 1-1 = 0 \to \sin 2\alpha = 0$

سوال ۶

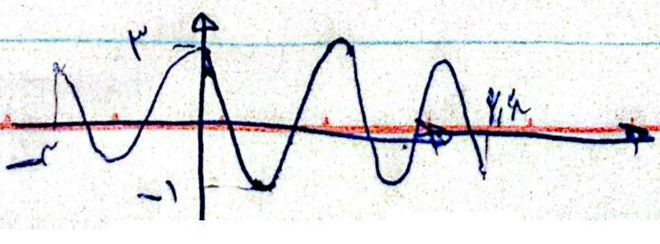
الف) $-2\sin x\cos x + 2 \to -2\times\frac{1}{2}\sin 2x + 2$

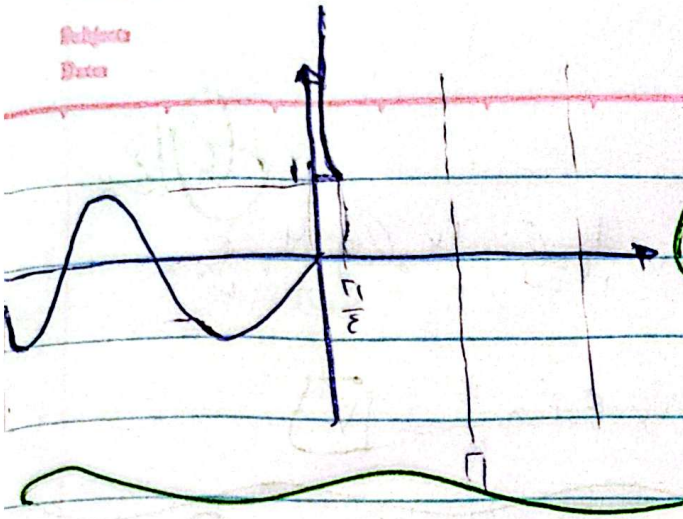
$2 - \sin 2x \to T = \frac{2\pi}{2} = \pi$



ب) $2\cos(x+\frac{1}{2})\pi + 1 \to 2\cos(\pi x + \frac{\pi}{2}) + 1 \to 1 - 2\sin(\pi x)$

$T = \frac{2\pi}{\pi} = 2$





سوال ⑦

$R_{f(x)} = (-1, +\infty)$

سوال ⑧

$f(x) = a + b\sin(\omega x)\cos(\omega x)\cos(2\omega x)$

$f(x) = a + \frac{b}{2}\sin(2\omega x)\cos(4\omega x)$

$f(x) = a + \frac{b}{4}\sin(4\omega x)$

$T = \frac{5\pi}{10} - \frac{\pi}{10} \Rightarrow \frac{4\pi}{5} = \frac{2\pi}{4C} \Rightarrow C = \frac{5}{2}$

$a + \frac{b}{4} = 4$

$a = 3$ $\quad b = \frac{2-\infty}{2} \Rightarrow 2$

$\frac{bc}{a} = \frac{\frac{5}{4} \times 2}{2} \Rightarrow \frac{5}{4}$

سوال ⑨

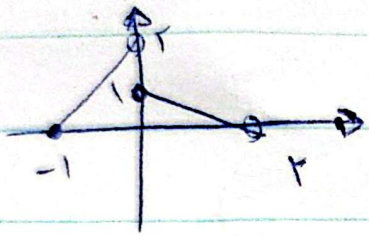
$a\cos bx + c$

$T = \frac{2\pi}{|b|} = \pi \longrightarrow |b| = 2 \longrightarrow b = \pm 2$

$c + |a| = 2$, $c - |a| = -2$ $\quad c = -1$, $a = 3$

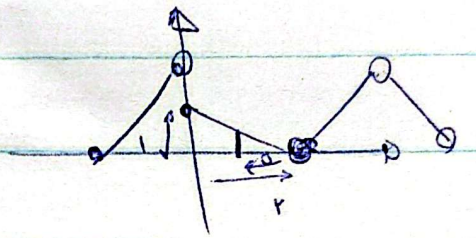
$3\cos(2x) - 1 \Rightarrow 3\cos(2x) - 1 = 0 \rightarrow \cos 2x = \frac{1}{3}$

$|\sin 2x| = 1 - \frac{1}{9} = \frac{\sqrt{8}}{3}$ $\quad |\tan x| \rightarrow \left|\frac{\frac{\sqrt{8}}{3}}{\frac{1}{3}}\right| \rightarrow \sqrt{8}$



$f(-۳\frac{۱}{۴},۰) = f(-۳\frac{۱}{۴}+۴,۰)$

$(-۱,۲۲) = -۳\frac{۱}{۴}$



۱	۲
$\frac{۳}{۴}$	$\frac{۱}{۲}$

$f(-۳\frac{۱}{۴},۰) = \boxed{\frac{۱}{۴}+}$