

$$\hat{B} + \hat{C} + \hat{A} = 180^\circ \rightarrow \hat{A} = 60^\circ \rightarrow \frac{BC}{\sin A} = \frac{AB}{\sin C} = \frac{AC}{\sin B} \rightarrow \frac{20}{\frac{\sqrt{3}}{2}} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}$$

$$\frac{AC}{\sin B} = \frac{20\sqrt{6}}{3} = \frac{40\sqrt{3}}{3} \rightarrow \sin B = \frac{20\sqrt{6}\sqrt{2}}{40\sqrt{3}} = \frac{\sqrt{2}}{2} \rightarrow B = 45^\circ, C = 180 - 45 = 75$$

$$\hat{A}_1 = 60^\circ, \hat{A}_2 = 45^\circ \rightarrow \frac{2\sqrt{2}}{\sin 45^\circ} = \frac{AC}{\sin 60^\circ} \rightarrow \frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}} = 4 = \frac{AC}{\frac{\sqrt{3}}{2}} \rightarrow AC = 2\sqrt{3}$$

مسئله ADC

$$AB \text{ (مجهول)} \rightarrow \frac{AC}{\sin 90^\circ} = \frac{x}{\sin 60^\circ} \rightarrow \frac{2\sqrt{3}}{1} = \frac{x}{\frac{\sqrt{3}}{2}} \rightarrow x = 4$$

$$(b^2 - 2) \frac{\sin C}{\sin B} = c \rightarrow \frac{b^2 - 2}{\sin B} = \frac{c}{\sin C} = \frac{b}{\sin B} \rightarrow b^2 - 2 = b$$

$$b^2 - b - 2 = 0 \rightarrow b = -1 \text{ (غیرممکن)}, b = 2 \checkmark$$

$$b^2 = a^2 + c^2 - 2ac \cos B \rightarrow 169 = 400 + 441 - 840 \cos B \rightarrow 840 \cos B = 672$$

$$\cos B = \frac{8}{10} \rightarrow \sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \frac{64}{100}} = \frac{6}{10}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 16 + 16 + 8\sqrt{3} - 8 - 8\sqrt{3} = 24 \rightarrow a = \sqrt{24} = 2\sqrt{6}$$

ب) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = 4\sqrt{2} = \frac{4}{\sin B} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow B = 45^\circ$

$$\hat{C} = 180^\circ - \hat{B} - \hat{A} = 180^\circ - 45^\circ - 60^\circ = 75^\circ$$

$$ABC \rightarrow BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos \alpha \rightarrow 4 = 4 + 1 - 2AB \cos \alpha \rightarrow \cos \alpha = \frac{1}{4}$$

$$ADE \rightarrow DE^2 = AD^2 + AE^2 - 2AD \cdot AE \cdot \cos \alpha = 16 + 36 - 12 = 40$$

$$DE = \sqrt{40} = 2\sqrt{10}$$

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$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos 60 = 64 + 25 - 40 = 49 \rightarrow BC = 7$$

$$S = \frac{1}{2} \times AB \times AC \times \sin 60 = \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

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$$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc \cos A}{bc(\cos A + 1)} = \frac{2bc + 2bc \cos A}{bc(\cos A + 1)} = 2$$

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$$\frac{1}{2} \times (3 - \cos \theta) \times (1 + 3 \cos \theta) \times \sin \theta = 2 \rightarrow (3 - \cos \theta)(1 + 3 \cos \theta) = 8$$

$$-3 \cos^2 \theta + 8 \cos \theta - 5 = 0 \rightarrow \Delta = 4 \rightarrow \cos \theta = \frac{-8 \pm 2}{-6} \rightarrow \cos \theta = \frac{1}{3} \text{ or } \frac{5}{3}$$

$$a^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos 30 = 4 + 16 - 8\sqrt{3} = 20 - 8\sqrt{3}$$

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