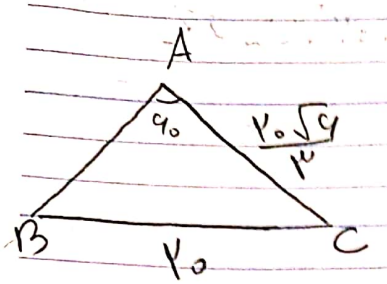
همساجای / تکلیف شماره‌ی ۱۳



① $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \rightarrow \frac{۲۰}{\frac{\sqrt{۳}}{۲}}=\frac{\frac{۲۰\sqrt{۹}}{۳}}{\sin B}$

$\frac{۴۰}{\sqrt{۳}}=\frac{۲۰}{۳\sin B} \Rightarrow \sin B=\frac{\sqrt{۱۸}}{۹}=\frac{\sqrt{۲}}{۲} \rightarrow B=۴۵^\circ$

$C=۱۲۰-۴۵=۷۵^\circ$

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② $A=۱۸۰-(۹۰+۷۵)+۹۰^\circ=۱۰۵^\circ \begin{cases}\rightarrow \hat{A}_1=۹۰^\circ \\ \rightarrow \hat{A}_2=۴۵^\circ\end{cases}$

$AC=\frac{۹k}{\sin ۹۰}=\frac{۲۹k}{\sqrt{۳}}$

$\frac{۲\sqrt{۲}}{\frac{\sqrt{۲}}{۲}}=\frac{\frac{۲۹k}{\sqrt{۳}}}{\frac{\sqrt{۳}}{۲}} \rightarrow ۴=\frac{۴۹k}{۳} \rightarrow k=۳$

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③ $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \rightarrow \frac{\sin C}{\sin B}=\frac{c}{b}$

$(b^۲-۲)\frac{k}{b}=۱ \rightarrow \frac{b^۲-۲}{b}=۱ \rightarrow b^۲-۲=b \rightarrow b^۲-b-۲=۰ \begin{cases}\rightarrow b=-۱ \times \\ \rightarrow b=۲ \checkmark\end{cases}$

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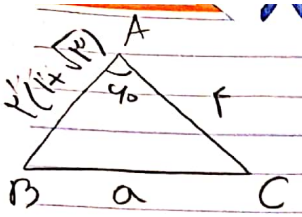
④ $۱۳^۲=۲۰^۲+۱۱^۲-۲(۲۰)(۱۱)\cos\hat{B} \rightarrow$

$۱۶۹=۴۰۰+۱۲۱-۸۴۰\cos\hat{B} \rightarrow ۸۴۰\cos\hat{B}=۳۵۲ \rightarrow$

$\cos\hat{B}=\frac{۳۵۲}{۸۴۰}=\frac{۴}{۵} \rightarrow \sin\hat{B}=\frac{۳}{۵}$



(ب)

$a^2 = 4^2 + (2(1+\sqrt{3}))^2 - 2(4)(2+2\sqrt{3})\cos 60^\circ$

$a^2 = 16 + 16 + 8\sqrt{3} - 8 - 8\sqrt{3} \rightarrow a^2 = 24$

$a = \sqrt{24} = 2\sqrt{6}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sin B} \rightarrow \sin B = \frac{2\sqrt{3}}{2\sqrt{6}} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ$

$\hat{C} = 180 - (45+60) = 75^\circ$

(ج)

$4 = \frac{4+1}{\omega} - 2(2)(1)\cos\alpha \rightarrow 4 = \omega - 4\cos\alpha \rightarrow$

$4\cos\alpha = 1 \rightarrow \cos\alpha = \frac{1}{4}$

$DE^2 = 16 + 36 - 2(4)(6)\cos\alpha = 16 + 36 - 12 = 40 \rightarrow DE = \sqrt{40} = 2\sqrt{10}$

(الف)

$BC^2 = 20 + 64 - 2(\omega)(8)\cos 60^\circ = 20 + 64 - 40 = 44$

$BC = 7$

(ب)

$S = \frac{1}{2} \times \omega \times 8^2 \times \sin 60^\circ = 10\sqrt{3}$

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)}$

$= 2$

۹

$$\frac{a^3+b^3-c^3}{a+b-c}=a^2$$

$$a^2=b^2+c^2-2bc\cos\hat{A}$$

$$a+(b-c)(b^2+bc+c^2)=a^3+a^2(b-c)\rightarrow$$

$$a^2=b^2+bc+c^2=b^2+c^2-2bc\cos\hat{A}\rightarrow bc=-2bc\cos\hat{A}$$

$$\rightarrow \cos\hat{A}=\frac{-1}{2}$$

۱۰

$$\frac{1}{2}\times(1+3\cos\theta)(3-\cos\theta)\sin^2 30=2$$

$$\frac{1}{2}$$

$$-3\cos^2\theta+8\cos\theta+3=8\rightarrow 3\cos^2\theta-8\cos\theta+5=0$$

$$\rightarrow \cos\theta=1\ \checkmark$$

$$\rightarrow \cos\theta=\frac{5}{3}\times$$

$$a^2=(\frac{3}{2}-1)^2+(1+\frac{3}{4})^2-2(2)(4)\cos 30=\frac{4+16}{4}-8\sqrt{3}=$$

$$\frac{\sqrt{3}}{2}$$

$$20-8\sqrt{3}$$