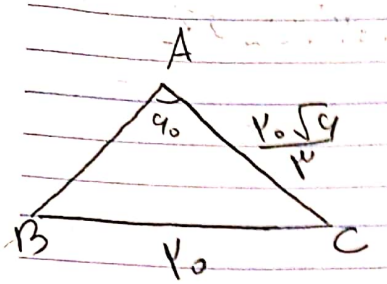
۲۰ آفرین

شنبه ۳۰ ۹ صفر ۱۴۳۷ 21 NOV 2015 آبان ۹۴

همسایگی / تکلیف شماره‌ی ۱۳



① ② $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \rightarrow \frac{۲۰}{\frac{\sqrt{۳}}{۲}}=\frac{۲۰\sqrt{۶}}{۳\sin B}$

$\frac{۴۰}{\sqrt{۳}}=\frac{۲۰\sqrt{۶}}{۳\sin B} \Rightarrow \sin B=\frac{\sqrt{۱۸}}{۶}=\frac{\sqrt{۲}}{۲} \rightarrow B=۴۵^\circ$ ✓

$C=۱۲۰-۴۵=۷۵^\circ$ ✓

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②

$A=۱۸۰-(۴۵+۷۵)+۴۵^\circ=۱۰۵^\circ \begin{cases} \rightarrow \hat{A}_1=۴۵^\circ \\ \rightarrow \hat{A}_2=۶۰^\circ \end{cases}$

② $AC=\frac{۹x}{\sin ۴۵}=\frac{۲۹x}{\sqrt{۲}}$

$\frac{۲\sqrt{۲}}{\sin ۶۰}=\frac{\frac{۲۹x}{\sqrt{۲}}}{\sin ۴۵} \rightarrow ۲=\frac{۶x}{۳} \rightarrow x=۳$ ✓

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③ ② $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \rightarrow \frac{\sin C}{\sin B}=\frac{c}{b}$

$(b^۲-۲)\frac{k}{b}=e \rightarrow \frac{b^۲-۲}{b}=۱ \rightarrow b^۲-۲=b \rightarrow b^۲-b-۲=۰ \begin{cases} \rightarrow b=-۱ \times \\ \rightarrow b=۲ \checkmark \end{cases}$

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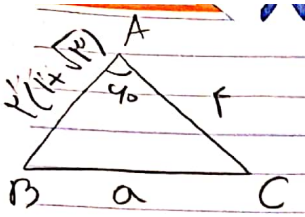
④

$۱۳^۲=۲۰^۲+۱۱^۲-۲(۲۰)(۱۱)\cos\hat{B} \rightarrow$

$۱۶۹=۴۰۰+۱۲۱-۸۴۰\cos\hat{B} \rightarrow ۸۴۰\cos\hat{B}=۳۵۲ \rightarrow$ ②

$\cos\hat{B}=\frac{۳۵۲}{۸۴۰}=\frac{۴}{۱۰} \rightarrow \sin\hat{B}=\frac{۳}{۱۰}$ ✓



(ب)

$a^2 = 4^2 + (2(1+\sqrt{3}))^2 - 2(4)(2+2\sqrt{3})\cos 45^\circ$ (۳)

$a^2 = 16 + 16 + 8\sqrt{3} - 8 - 8\sqrt{2} \rightarrow a^2 = 24$ (۲)

$a = \sqrt{24} = 2\sqrt{6}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{4}{\sin B} \rightarrow \sin B = \frac{4\sqrt{3}}{4\sqrt{6}} = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ$ (۲)

$\hat{C} = 180 - (45+45) = 90^\circ$

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$4 = \frac{4+1}{\omega} - 2(2)(1)\cos\alpha \rightarrow 4 = \omega - 4\cos\alpha \rightarrow$ (۲)

$4\cos\alpha = 1 \rightarrow \cos\alpha = \frac{1}{4}$

$DE^2 = 16 + 9 - 2(4)(3)\cos\alpha = 16 + 9 - 12 = 13 \rightarrow DE = \sqrt{13} = 2\sqrt{10}$

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(الف

$BC^2 = 25 + 49 - 2(5)(7)\cos 45^\circ = 25 + 49 - 70 = 49$

$BC = 7$ (۲)

(ب

$S = \frac{1}{2} \times 5 \times 7^2 \times \sin 45^\circ = 70\sqrt{2}$ (۲)

۸

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$ (۲)

$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2 + c^2 + 2bc - b^2 - c^2 + 2bc\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)}$

$= 2$

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$$\frac{a^3+b^3-c^3}{a+b-c}=a^2$$

$$a^2=b^2+c^2-2bc\cos\hat{A}$$

$$a+(b-c)(b^2+bc+c^2)=a^3+a^2(b-c)\rightarrow$$

$$a^2=b^2+bc+c^2=b^2+c^2-2bc\cos\hat{A}\rightarrow bc=-2bc\cos\hat{A}$$

$$\rightarrow \cos\hat{A}=-\frac{1}{2}$$

۱۰

$$\frac{1}{2}\times(1+3\cos\theta)(3-\cos\theta)\underbrace{\sin^2 30}_{\frac{1}{2}}=2$$

$$-3\cos^2\theta+8\cos\theta+3=8\rightarrow 3\cos^2\theta-8\cos\theta+5=0$$

$$\rightarrow \cos\theta=1 \checkmark$$

$$\rightarrow \cos\theta=\frac{5}{3}\times$$

$$a^2=(\underbrace{3-1}_{2})^2+(\underbrace{1+3}_{4})^2-2(2)(4)\underbrace{\cos 30}_{\frac{\sqrt{3}}{2}}=4+16-8\sqrt{3}=$$

$$20-8\sqrt{3}$$