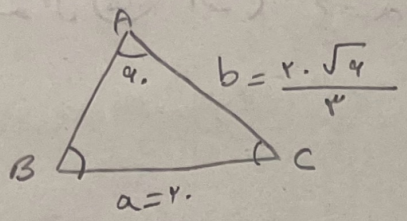
پارسین زندی

سؤال ①



$\hat{B}+\hat{C}=۱۲۰$

$\hat{A}=۶۰$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$\frac{۲۰}{\sin ۶۰}=\frac{۲۰\sqrt{۶}}{۳}$

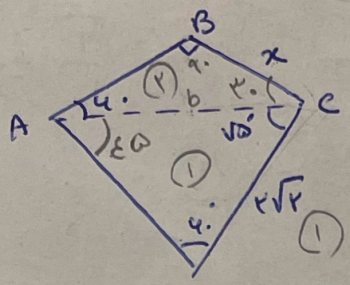
$\sin B=\frac{۱۰\cancel{۲۰}\sqrt{۶}}{\cancel{۳}}\times\frac{\sqrt{۲}}{\cancel{۲۰}}=$

$\hat{A}=۶۰$ | $\hat{B}=۴۵$ | $\hat{C}=۱۲۰-۴۵=۷۵$

$=\frac{۱۰\sqrt{۱۸}}{۳}=\frac{\cancel{۱۰}\sqrt{۱۸}}{\cancel{۲۰}\times۲}=\frac{\sqrt{۱۸}}{۶}=\frac{\cancel{۳}\sqrt{۲}}{\cancel{۶}}=\frac{\sqrt{۲}}{۲}$

$\Rightarrow \hat{B}=۴۵$

سؤال ②



$۶۰+۷۵=۱۳۵$

$۱۸۰-۱۳۵=۴۵$

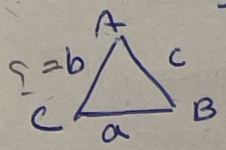
① $\frac{۲\sqrt{۲}}{\sin ۴۵}=\frac{b}{\sin ۶۰}\Rightarrow\frac{۲\sqrt{۲}}{\frac{\sqrt{۲}}{۲}}=\frac{b}{\frac{\sqrt{۳}}{۲}}\Rightarrow b=\frac{\cancel{۲}\sqrt{۲}\times\sqrt{۳}}{\cancel{۲}}\div\frac{\sqrt{۲}}{۲}=\frac{\sqrt{۶}}{\frac{\sqrt{۲}}{۲}}=۲\sqrt{۳}$

② $\frac{۲\sqrt{۳}}{\sin ۶۰}=\frac{x}{\sin ۶۰}\Rightarrow\frac{۲\sqrt{۳}}{۱}=\frac{x}{\frac{\sqrt{۳}}{۲}}\Rightarrow x=\cancel{۲}\sqrt{۳}\times\frac{\sqrt{۳}}{\cancel{۲}}=\boxed{۳}$

سؤال ③

$(b^۲-۲)\frac{\sin\hat{C}}{\sin\hat{B}}=c$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=K$



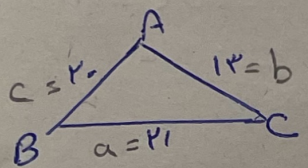
$\Rightarrow \sin C=\frac{c}{K}$, $\sin B=\frac{b}{K}$

$\frac{\sin C}{\sin B}=\frac{c}{b^۲-۲}\Rightarrow\frac{\frac{c}{K}}{\frac{b}{K}}\Rightarrow\frac{c}{b}=\frac{c}{b^۲-۲}\Rightarrow b^۲-b-۲=۰$

$\Rightarrow b=-۱$ ✗

$\boxed{b=۲}$ ✓

سؤال ④



$\sin\hat{B}=?$

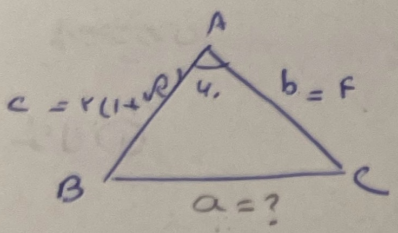
$b^۲=a^۲+c^۲-۲ac\cos B\Rightarrow(۱۴)^۲=(۲۱)^۲+(۲۰)^۲-۲\times۲۱\times۲۰\cos B$

$۱۹۶=۴۴۱+۴۰۰-۸۴۰\cos B$

$۸۴۰\cos B=۸۴۱-۱۹۶\Rightarrow\cos\hat{B}=\frac{۸۴۵}{۹۷۲}$

$\cos^۲B+\sin^۲B=۱\Rightarrow\boxed{\sin B=۰/۴}$

سؤال ⑤ الف) $a=?$ ب) $\hat{B}=?$ ، $\hat{C}=?$



$a^2=b^2+c^2-2bc\cos\hat{A} \Rightarrow a^2=(4)^2+(2(1+\sqrt{3}))^2-2bc\cos 60^\circ$

$(2+2\sqrt{3})^2$

$\Rightarrow a^2=14+(4+12+8\sqrt{3})-\frac{2\times(4)(2+2\sqrt{3})}{(14+14\sqrt{3})}\cos 60^\circ = 26+8\sqrt{3}-(14+14\sqrt{3})\times\frac{1}{2}=$

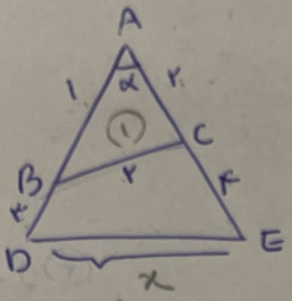
$8+8\sqrt{3}$

$26+8\sqrt{3}-8-8\sqrt{3}=24 \Rightarrow \boxed{a=\sqrt{24}}$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \Rightarrow \frac{\sqrt{24}}{\frac{\sqrt{3}}{2}}=\frac{4}{\sin B} \Rightarrow \sin B=\frac{4\times\frac{\sqrt{3}}{2}}{\sqrt{24}}=\frac{2\sqrt{3}}{2\sqrt{6}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{\sqrt{2}}{2} \Rightarrow \boxed{\hat{B}=45^\circ}$ $\quad \hat{C}=180-(60+45)=75 \Rightarrow \boxed{C=75}$

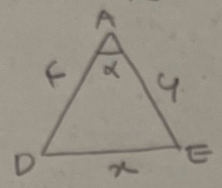
سؤال ⑥



① در مثلث $BC^2=AC^2+AB^2-2(AB)(AC)\cos A$

$4=4+1-4\cos\alpha \Rightarrow -1=-4\cos\alpha \Rightarrow$

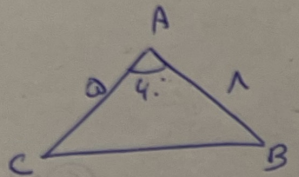
$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{4+1-4}{4} \Rightarrow \boxed{\cos A=\frac{1}{4}}$



$x^2=25+16-2(4)(5)\times\cos A=41-40\times\frac{1}{4}=41-10=31 \Rightarrow$

$\boxed{x=\sqrt{40}} \Rightarrow \boxed{x=2\sqrt{10}}$

سؤال ⑦

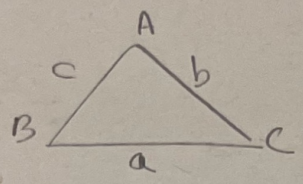


الف) $BC=?$ ب) $S_{\triangle ABC}=?$

$BC^2=25+64-2(40)\cos 60^\circ=89-40=49 \Rightarrow \boxed{BC=7}$

$S_{\triangle ABC}=\frac{1}{2}\times AC\times AB\times\sin 60^\circ=\frac{1}{2}\times 5\times 8\times\frac{\sqrt{3}}{2} \Rightarrow \boxed{S=10\sqrt{3}}$

سؤال ۸

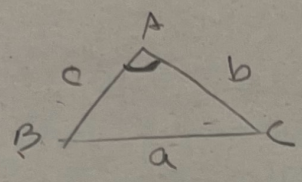


$$\frac{b^2+c^2+2bc-a^2}{bc+bc\cos\hat{A}} \qquad \frac{(b+c)^2-a^2}{bc(\cos\hat{A}+1)} = ?$$

$$\frac{b^2+c^2+2bc-b^2-c^2+2bc\cos A}{bc(1+\cos A)} = \frac{2bc(1+\cos A)}{bc(1+\cos A)} = \boxed{2}$$

سؤال ۹

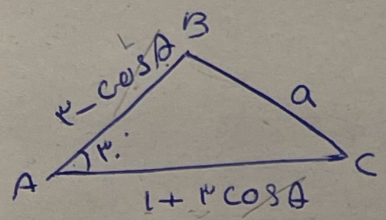
$$\frac{a^2+b^2-c^2}{a+b-c} = a^2$$



$\cos\hat{A} = ?$

$$\frac{a^2+b^2-c^2}{a+b-c} = b^2+c^2-2bc\cos\hat{A} \Rightarrow \frac{a^2+b^2-c^2}{a+b-c} - (b^2+c^2) = -2bc\cos\hat{A}$$

سؤال ۱۰



$S = 2$ $\quad a^2 = ?$

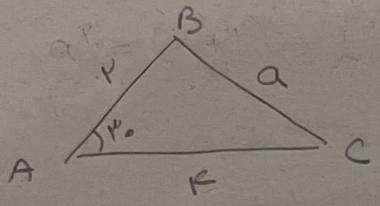
$$S = \frac{1}{2}\times AB\times AC\times \sin 60^\circ$$

$$S = \frac{1}{2}\times(2-\cos\theta)(1+2\cos\theta)\times\frac{1}{2}$$

$$2 = \frac{1}{4}(2+4\cos\theta-\cos\theta-2\cos\theta) \Rightarrow$$

$$\Rightarrow 2 = \frac{1}{4}(2+3\cos\theta) \Rightarrow 2 = \frac{2}{4}+\frac{3}{4}\cos\theta \Rightarrow \frac{3}{4} = \frac{3}{4}\cos\theta \Rightarrow \boxed{\cos\theta = 1}$$

$$\Rightarrow \boxed{\theta = 0}$$



$$a^2 = 16+4-2(4)(2)\cos 60^\circ \Rightarrow a^2 = 20-16\left(\frac{\sqrt{2}}{2}\right) = 20-8\sqrt{2}$$

$$\Rightarrow \boxed{a^2 = 20-8\sqrt{2}}$$