سارینا عابدینی

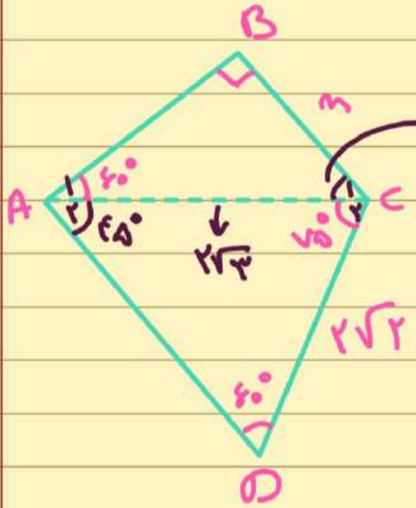
۱-

$$\hat{B}+\hat{C}=۱۲۰^\circ \xrightarrow{\hat{A}+\hat{B}+\hat{C}=۱۸۰^\circ} \hat{A}=۶۰^\circ$$

$$\frac{\sin ۶۰}{۲۰}=\frac{\sin\hat{B}}{\frac{۲۰\sqrt{۲}}{۳}} \qquad \left(\sin ۶۰=\frac{\sqrt{۳}}{۲}\right)$$

$$\sin\hat{B}=\frac{\sqrt{۲}}{۲}\to \hat{B}=۴۵^\circ \xrightarrow{B+C=۱۲۰^\circ} C=۷۵$$

۲-



$$۳۰^\circ$$

$$\xrightarrow{A_1+B+C_1=۱۸۰^\circ} C_1=۶۰^\circ$$

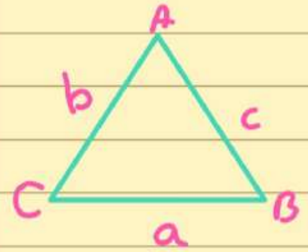
$$\xrightarrow{A_2+C_2+D=۱۸۰^\circ} A_2=۴۵^\circ$$

$$\frac{DC}{\sin A_2}=\frac{AC}{\sin D} \qquad (DC=۲\sqrt{۲},\ \sin A_2=\sin ۴۵^\circ,\ \sin D=\sin ۶۰^\circ)$$

$$\Longrightarrow \frac{۲\sqrt{۲}}{\frac{\sqrt{۲}}{۲}}=\frac{۲\sqrt{۳}}{\frac{\sqrt{۳}}{۲}}$$

$$AC\times\sin ۶۰^\circ = x \to ۲\sqrt{۳}\times\frac{\sqrt{۳}}{۲}=③$$

۳-



$$(b^2-۲)\frac{\sin\hat{C}}{\sin\hat{B}}=c$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\Downarrow$$

$$b\frac{\sin C}{\sin B}=c \Rightarrow b^2-۲=b \to b^2-b-۲=0$$

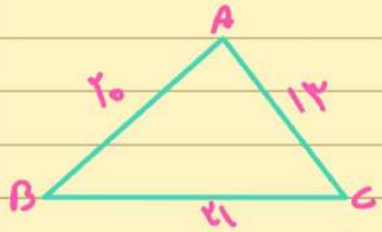
$a+c=b$ ، یکی از ریشه‌ها -۱ یکی از ریشه‌ها $-\frac{c}{a}$

$$b=-۱ \ , \ b=۲$$

★ اندازه‌ی ضلع نمی‌تواند منفی باشد پس: $b=۲$

سارینا عابدینی

۴-



$a^2=b^2+c^2-2bc\cos\hat{A}$

$169=400+441-840(\cos\hat{B})$

$\cos\hat{B}=\frac{672}{840}=\frac{4}{5}$

$\sin^2\hat{B}+\cos^2\hat{B}=1$

$\sin^2\hat{B}+\frac{16}{25}=1 \Longrightarrow \sin^2 B=\frac{9}{25} \Rightarrow \sin\hat{B}=\frac{3}{5}$

۵-

الف) $a^2=16+4(1+3+2\sqrt{3})-\left(16(1+\sqrt{3})\frac{1}{2}\right)$

$a^2=16+4+12+8\sqrt{3}-8-8\sqrt{3}=24 \Longrightarrow a=2\sqrt{6}$

ب) $\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}} \Longrightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}}=\frac{4}{\sin\hat{B}}$

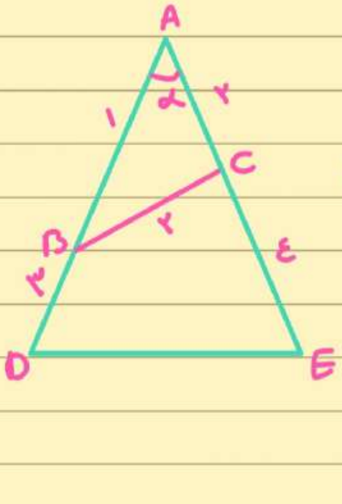
$\Downarrow$

$\sin\hat{B}=\frac{\sqrt{2}}{2}$

$\hat{B}=45^\circ$

$\xrightarrow{\hat{A}+\hat{B}+\hat{C}=180^\circ} \hat{C}=75^\circ$

۶-



$a^2=b^2+c^2-2bc\cos\hat{A}$

$4=4+1-4\cos\alpha \Rightarrow \cos\alpha=\frac{1}{4}$

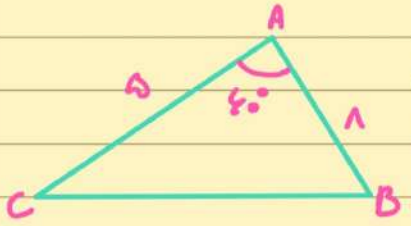
$(DE)^2=36+16-48(\cos\alpha)$ ($\cos\alpha=\frac{1}{4}$)

$(DE)^2=40 \rightarrow DE=2\sqrt{10}$

سارینا عابدینی

۷-

الف)



$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$(BC)^2 = ۶۴ + ۲۵ - ۸۰\left(\frac{1}{2}\right) \Rightarrow (BC)^2 = ۴۹ \Rightarrow BC = ۷$

ب)

$S_\Delta = \frac{1}{2} bc \sin\hat{A} \longrightarrow \frac{1}{2} \times ۸ \times ۵ \times \frac{\sqrt{۳}}{2} = ۱۰\sqrt{۳}$

۸-

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc - a^2 - b^2 + 2bc\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2bc(\cos\hat{A}+1)}{bc(\cos\hat{A}+1)} = ۲$$

$2bc + 2bc\cos\hat{A}$

۹-

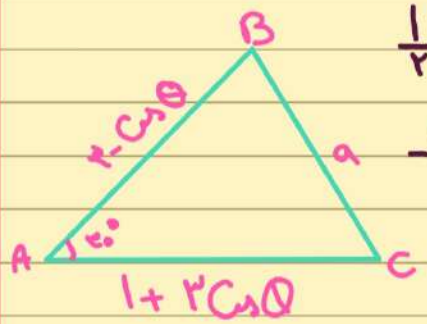
$$\frac{a^3 + b^3 - c^3}{a+b-c} = a^2 \longrightarrow a^3 + b^3 - c^3 = a^3 + a^2b - a^2c$$

$$b^3 - c^3 = a^2b - a^2c \xrightarrow{a \neq b \neq c} (b-c)(b^2+c^2-bc) = a^2(b-c)$$

$$a^2 = b^2 + c^2 - bc = b^2 + c^2 - 2bc\cos\hat{A}$$

$$2\cos\hat{A} = 1 \rightarrow \cos\hat{A} = \frac{1}{2}$$

۱۰-



$$\frac{1}{2} \times (1+3\cos\theta)(3-\cos\theta) \times \frac{1}{2} = ۲$$

$$-3\cos\theta + ۸\cos\theta - ۵ = 0 \xrightarrow[m_1=1, m_2=\frac{c}{a}]{a+b+c=0}$$

$\cos\theta = 1$

غ ق ق $-1 \leq \cos\theta \leq 1$

$\cos\theta = \frac{۵}{۳}$ ✗

$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$a^2 = ۱۶ + ۴ - ۸\sqrt{۳}$

$a^2 = ۲۰ - ۸\sqrt{۳}$