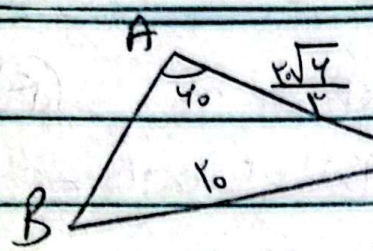
حل تمرین

Subject: Date: / /



$\hat{B}+\hat{C}=۱۳۵ \rightarrow \hat{A}=۴۵$ (۱)

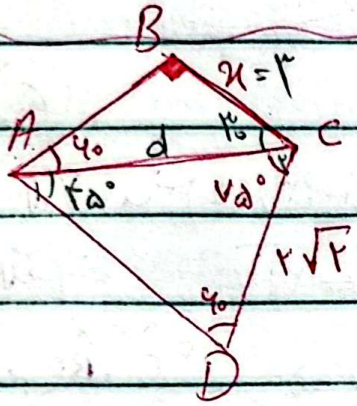
$\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}} \Rightarrow \frac{۲۰}{\frac{\sqrt{۲}}{۲}}=\frac{۲\sqrt{۲}}{\sin\hat{B}}$

$\frac{\sqrt{۲}}{۲}\times\frac{۲\sqrt{۲}}{۲}\times\frac{۱}{۲۰}=\frac{۲\times۲\sqrt{۲}}{۲\times۲\times۲۰}=\frac{\sqrt{۲}}{۲} \rightarrow \sin۴۵^\circ \checkmark \rightarrow \hat{B}=۴۵^\circ$

$\sin۱۳۵ \rightarrow ۱۳۵+۴۵<۱۸۰$

یعنی $۱۹۵<۱۸۰$

$۴۵+۴۵=۱۰۵ \rightarrow ۱۸۰-۱۰۵=\hat{C}=۷۵$

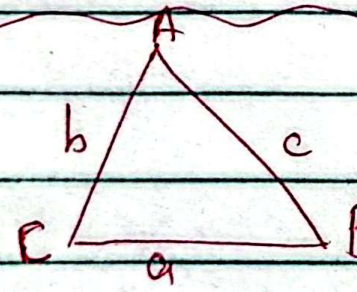


$۱۸۰-(۴۵+۷۵)=۶۰$ (۲)

$\frac{a}{\sin\hat{A}_1}=\frac{d}{\sin\hat{D}}=\frac{c}{\sin\hat{C}_1} \Rightarrow \frac{۲\sqrt{۲}}{\frac{\sqrt{۳}}{۲}}=\frac{d}{\frac{\sqrt{۳}}{۲}}$

$d=\frac{\sqrt{۳}}{۲}\times۲\sqrt{۲}\times\frac{۲}{\sqrt{۳}}=\boxed{۲\sqrt{۲}=d}$

$\sin۶۰^\circ=\frac{\sqrt{۳}}{۲}=\frac{x}{۲\sqrt{۳}} \rightarrow \frac{۲\sqrt{۳}\times\sqrt{۳}}{۲}=۳=x$



(۳)

$(b^2-۲)\left(\frac{\sin C}{\sin B}\right)=c$

$\frac{a}{\sin\hat{A}}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}} \rightarrow c=\frac{\sin\hat{C}\times b}{\sin\hat{B}}$

$(b^2-۲)\left(\frac{\sin\hat{C}}{\sin\hat{B}}\right)=\frac{\sin\hat{C}\times b}{\sin\hat{B}} \Rightarrow b^2-۲=\frac{\sin C\times b}{\sin B}\times\frac{\sin B}{\sin C}=b$

$b^2-۲=b \rightarrow b^2-b-۲=۰ \rightarrow (b-۲)(b+۱)=۰ \Rightarrow$

$b=۲$ یا $b=-۱$ غیرقابل قبول چون طول ضلع مثلث نمی‌تواند منفی باشد

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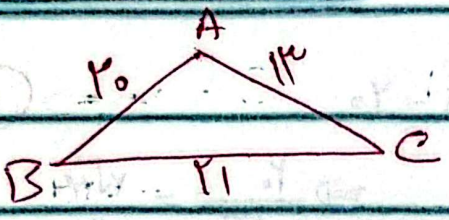
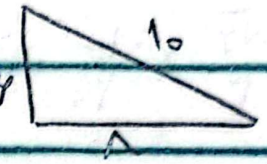


$b^2 = a^2 + c^2 - 2ac\cos\hat{B} \Rightarrow$ (۴)

$(۱۴)^2 = (۲۰)^2 + (۲۱)^2 - ۲\times۲۱\times۲۰\cos\hat{B} \Rightarrow$

$۱۹۶ = ۴۰۰ + ۴۴۱ - ۸۴۰\cos\hat{B} \Rightarrow ۱۹۶ = ۸۴۱ - ۸۴۰\cos\hat{B} \Rightarrow$

$۶۷۲ = ۸۴۰\cos\hat{B} \Rightarrow \frac{۶۷۲}{۸۴۰} = \frac{۸}{۱۰} \rightarrow$



$\sin\hat{B} = \frac{۶}{۱۰}$

$b=۴$, $c=۲(۱+\sqrt{۳})$ $\hat{A}=۶۰^\circ$ (۵)

$a^2 = b^2 + c^2 - ۲bc\cos\hat{A} \Rightarrow ۱۶ + ۴(۱+۳+۲\sqrt{۳}) - ۱۶(۱+\sqrt{۳})(\frac{۱}{۲})$ اندازه a را بیابید

$۱۶ + ۴ + ۱۲ + ۸\sqrt{۳} - ۸ - ۸\sqrt{۳} = ۲۴ \rightarrow a = \sqrt{۲۴} = ۲\sqrt{۶}$ (ب)

$\frac{b}{\sin\hat{B}} = \frac{a}{\sin\hat{A}} = \frac{c}{\sin\hat{C}} \Rightarrow \frac{۲\sqrt{۶}}{\frac{\sqrt{۳}}{۲}} = \frac{۴}{\sin\hat{B}} = \frac{۲+۲\sqrt{۳}}{\sin\hat{C}} \Rightarrow$ اندازه $\hat{C}$ و $\hat{B}$ را بیابید

$\frac{۴}{۱}\times\frac{۲\sqrt{۳}}{۳}\times\frac{۱}{۲\sqrt{۶}} = \frac{\sqrt{۳}}{\sqrt{۶}} \rightarrow \frac{۴\sqrt{۲}}{۶} = \frac{\sqrt{۲}}{۲} = \sin\hat{B} \Rightarrow$ $\hat{B}=۴۵^\circ$

$\hat{B} = ۱۳۵^\circ$ ق ق ق

$۱۸۰ - (۴۵ + ۷۵) = ۷۵ = \hat{C}$

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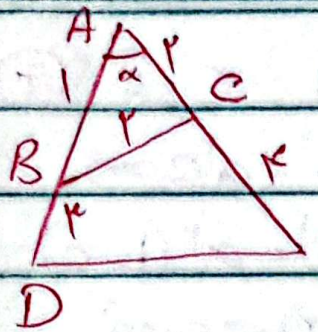


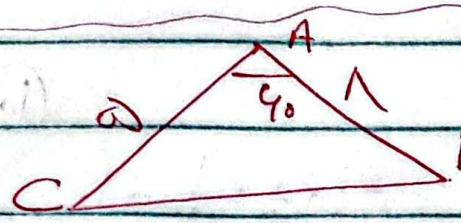
$(BC)^2 = (AB)^2 + (AC)^2 - 2(AC)(AB)\cos\alpha \Rightarrow$ (۶)

$4 = 4 + 1 - 2\times2\times1\times\cos\alpha \Rightarrow 4 = 5 - 4\cos\alpha$

$+1 = 4\cos\alpha \rightarrow \frac{1}{4} = \cos\alpha$

$(DE)^2 = (AD)^2 + (AE)^2 - 2(AE)(AD)\cos\alpha \Rightarrow 16 + 36 - 48\times\frac{1}{4} \Rightarrow$

$25 - 12 = (13) = DE^2 \rightarrow DE = \sqrt{13}$



محیط $\triangle ABC$ (۷)

$(CB)^2 = (AC)^2 + (AB)^2 - 2(AC)(AB)\cos\hat{A}$

$\Rightarrow 25 + 64 - 40 = 49 \Rightarrow CB = 7$

مساحت $\triangle ABC$

$S_{ABC} = \frac{1}{2}\times AC\times AB\times\sin\hat{A} = \frac{1}{2}\times5\times8\times\frac{\sqrt{3}}{2} = 10\sqrt{3}$

$\frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc\cos\hat{A})}{bc\cos A + bc} \Rightarrow$ (۸)

$\frac{2bc + 2bc\cos\hat{A}}{bc + bc\cos\hat{A}} \Rightarrow \frac{2(bc + bc\cos\hat{A})}{bc + bc\cos\hat{A}} = 2$ (۲)

(4) $\cos\hat{A} = ?$

$$\frac{a^3+b^3-c^3}{a+b-c} = a^2 \Rightarrow a^3+b^3-c^3 = a^3 + ba^2 - a^2c \Rightarrow$$

$$b^3 - c^3 = ba^2 - a^2c \rightarrow (b-c)(b^2+c^2+bc) = a^2(b-c)$$

$$\Rightarrow b^2+c^2+bc = a^2 \rightarrow a^2 = b^2+c^2-2bc\cos\hat{A}$$

$$-2bc\cos\hat{A} \rightarrow bc \Rightarrow \cos\hat{A} = -\frac{1}{2} \rightarrow 120^\circ = \hat{A}$$

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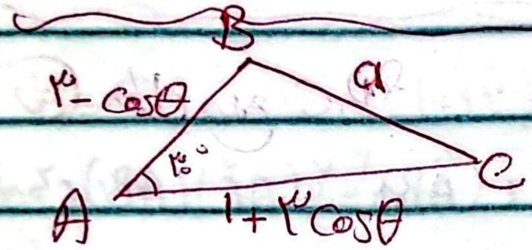
(5)



$S_{ABC} = 2$

$$(3-\cos\theta)(1+3\cos\theta) \times \frac{1}{2} \times \frac{1}{2} = 2$$

$$8 = 3 + 9\cos\theta - \cos\theta - 3\cos^2\theta \Rightarrow 3\cos^2\theta - 8\cos\theta + 5$$

$$\cos\theta = \frac{+8 \pm \sqrt{64-60}}{6} \rightarrow \frac{+8+2}{6} = \frac{10}{6} = \frac{5}{3}$$

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$$\frac{+8-2}{6} = \frac{6}{6} = 1 = \cos\theta$$

$$a^2 = (\underbrace{3-\cos\theta}_{2})^2 + (\underbrace{1+3\cos\theta}_{4})^2 - 2(\underbrace{3-\cos\theta}_{2})(\underbrace{1+3\cos\theta}_{4}) \times \frac{\sqrt{3}}{2}$$

$$a^2 = 4 + 16 - 8\sqrt{3} \rightarrow a^2 = 20 - 8\sqrt{3}$$