

$$\begin{aligned} BC = a \quad AC = b \quad \hat{B} + \hat{C} = 120^\circ \rightarrow \hat{A} = 60^\circ \quad \frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{20}{\frac{\sqrt{3}}{2}} = \frac{20\sqrt{3}}{\sin B} \quad \sin B = \frac{\sqrt{3}}{2} \\ \hat{B} = 60^\circ \quad \hat{C} = 120^\circ - 60^\circ = 60^\circ \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{AC}{\sin 60^\circ} = \frac{CD}{\sin A} \quad A = 180^\circ - (60^\circ + 60^\circ) = 60^\circ \quad \frac{AC}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} \quad AC = 2\sqrt{3} \quad x = \frac{\sqrt{3}}{2} \times AC \\ x = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3 \end{aligned} \quad (2)$$

$$\begin{aligned} (b^2 - 2) \frac{\sin C}{\sin B} = c \rightarrow \frac{\sin C}{c} = \frac{\sin B}{b^2 - 2} \quad \text{قضیه سینوس ها} \quad \frac{\sin C}{c} = \frac{\sin B}{b} \quad b^2 - 2 = b \\ b^2 - b - 2 = 0 \quad \begin{cases} b = -1 \quad \text{GGG} \\ b = 2 \end{cases} \end{aligned} \quad (3)$$

$$\begin{aligned} AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos B \quad 199 = 500 + 441 - 2 \times 20 \times 21 \times \cos B \quad \cos B = \frac{19}{20} = \frac{19}{20} \\ \sin B = \frac{21}{20} \end{aligned} \quad (4)$$

$$\begin{aligned} \text{الف) } a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad a^2 = 19 + 19 + 2\sqrt{3} - 2(2 + 2\sqrt{3}) \times 5 \times \frac{1}{2} \quad a^2 = 19 + 19 + 2\sqrt{3} - 10 - 10\sqrt{3} \\ a^2 = 28 \rightarrow a = 2\sqrt{7} \\ \text{ب) } \frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{2\sqrt{7}}{\frac{\sqrt{3}}{2}} = \frac{5}{\sin B} \quad \sin B = \frac{\sqrt{3}}{2} \quad B = 60^\circ \quad C = 180^\circ - (60^\circ + 60^\circ) = 60^\circ \end{aligned} \quad (5)$$

$$\begin{aligned} \triangle ABC: BC^2 = AC^2 + AB^2 - 2AC \times AB \times \cos \alpha \quad 5 = 5 + 1 - 2 \times 2 \times 1 \times \cos \alpha \quad \cos \alpha = \frac{1}{2} \\ \triangle ADE: DE^2 = AE^2 + AD^2 - 2AE \times AD \times \cos \alpha \quad DE^2 = 39 + 19 - 2 \times 9 \times 5 \times \frac{1}{2} \quad DE^2 = 80 \rightarrow DE = 4\sqrt{5} \end{aligned} \quad (6)$$

$$\begin{aligned} \text{الف) } BC^2 = AB^2 + AC^2 - 2AB \times AC \times \cos A \quad BC^2 = 9 + 16 - 2 \times 3 \times 4 \times \frac{1}{2} \quad BC^2 = 13 \rightarrow BC = \sqrt{13} \\ \text{ب) } S_{ABC} = \frac{1}{2} AB \times AC \times \sin A = \frac{1}{2} \times 3 \times 4 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \end{aligned} \quad (7)$$

$$\frac{b^2 + c^2 + 2bc - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cdot \cos A)}{bc(\cos A + 1)} = \frac{2bc + 2bc \cdot \cos A}{bc(\cos A + 1)} = \frac{2bc(1 + \cos A)}{bc(\cos A + 1)} = 2 \quad (8)$$

$$\begin{aligned} a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c \quad b^2 - c^2 = a^2 b - a^2 c \quad (b - c)(b^2 + bc + c^2) = a^2(b - c) \\ \left. \begin{aligned} a^2 &= b^2 + bc + c^2 \\ a^2 &= b^2 + c^2 - 2bc \cdot \cos A \end{aligned} \right\} \rightarrow bc = -2bc \cdot \cos A \quad \cos A = -\frac{1}{2} \end{aligned} \quad (9)$$

$$\begin{aligned} S = \frac{1}{2} \times AB \times AC \times \sin A \quad \frac{1}{2} \times (3 - \cos \theta)(1 + 3 \cos \theta) \times \frac{1}{2} = 2 \quad -3 \cos^2 \theta + 4 \cos \theta - 1 = 0 \\ 3 \cos^2 \theta - 4 \cos \theta + 1 = 0 \quad (\cos \theta - \frac{1}{3})(\cos \theta - 1) = 0 \quad \begin{cases} \cos \theta = \frac{1}{3} \quad \text{GGG} \\ \cos \theta = 1 \end{cases} \quad \begin{aligned} AB &= 2 \\ AC &= 5 \end{aligned} \\ a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad a^2 = 19 + 5 - 2 \times 2 \times 5 \times \frac{\sqrt{3}}{2} \quad a^2 = 10 - 10\sqrt{3} \end{aligned} \quad (10)$$