

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{r}{\sqrt{r}} = \frac{r\sqrt{r}}{\sin B} \rightarrow \sin B = \frac{\sqrt{r}}{r} \rightarrow \hat{B} = 45^\circ, \hat{C} = 90^\circ \quad (1)$$

$$17. - (4 + \sqrt{5}) = 45 \rightarrow \frac{r\sqrt{r}}{\sin 45} = \frac{AC}{\sin 45} \rightarrow AC = \frac{\sqrt{r} \times r\sqrt{r}}{\frac{\sqrt{2}}{2}} = r\sqrt{r} \quad (2)$$

$$\frac{x}{\sin 45} = \frac{r\sqrt{r}}{\sin 45} \rightarrow x = r\sqrt{r} \times \frac{\sqrt{2}}{r} = \sqrt{2}$$

$$\frac{\sin \hat{B}}{b-r} = \frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \rightarrow b-r = b \rightarrow b-r-r=0 \rightarrow b=1 \times \quad (3)$$

$$b^2 = a^2 + c^2 - 2ac \cos B \rightarrow 179 = 41 + 2 - 2(1)(r) \times \cos B \rightarrow \cos B = \frac{1}{r} \rightarrow \sin B = \frac{\sqrt{r}}{r} \quad (4)$$

$$a) a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 17 + 17 + 8\sqrt{r} - 2(4)(r) \times \frac{1}{r} = 17 \rightarrow a = \sqrt{17} \quad (5)$$

$$b) \frac{\sin B}{b} = \frac{\sin A}{a} \rightarrow \sin B = \frac{\sqrt{r}}{r} \rightarrow \hat{B} = 45^\circ, \hat{C} = 90^\circ$$

$$BC^2 = AC^2 + AB^2 - 2(AC)(AB) \cos A \rightarrow 42 = 1 + 2 - 2(1)(1) \times \cos A \rightarrow \cos A = \frac{1}{2} \quad (6)$$

$$DE^2 = 17 + 17 - 2(4)(1) \times \frac{1}{2} \rightarrow DE = \sqrt{2} = \sqrt{2}$$

$$a) BC^2 = 42 + 17 - 2(1)(1) \times \frac{1}{2} \rightarrow BC = \sqrt{5} \quad (7)$$

$$b) S_{ABC} = \frac{1}{2} \times 1 \times 1 \times \sin 45 = \frac{1}{2} \sqrt{2}$$

$$\frac{b^2 + c^2 + rbc - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + rbc - (b^2 + c^2 - 2bc \cos \hat{A})}{bc(\cos \hat{A} + 1)} = \frac{rbc(1 + \cos \hat{A})}{bc(1 + \cos \hat{A})} = r \quad (8)$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a \rightarrow a^2 + b^2 - c^2 = a^2 + ab - ac \rightarrow b^2 - c^2 = a(b-c) \quad (9)$$

$$\rightarrow (b-c)(b^2 + bc + c^2) = a(b-c) \xrightarrow{b \neq c} b^2 + bc + c^2 = a \rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$\rightarrow bc = -2bc \cos A \rightarrow \cos A = \frac{-1}{r}$$

$$\frac{1}{r}(r - \cos A)(1 + r \cos A) < \frac{1}{r} = 1 \rightarrow r \cos A - 1 \cos A + 1 = 1 \rightarrow \cos A = 1 \quad (10)$$

$$\cos A = 1 \rightarrow b = 1 + r = 4, c = r - 1 = 2$$

$$a^2 = c^2 + b^2 - 2bc \cos A = 4 + 17 - 2(2)(4) \times \frac{1}{2} = 17$$