

۱۹، ۷۵ آفرین

شماره تکلیف: ۱۳۰، تاریخ: روز دوشنبه ۱۳۰۰

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{r}{\sin 45^\circ} = \frac{r\sqrt{2}}{\sin B} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow \hat{B} = 45^\circ, \hat{C} = 90^\circ \quad (1)$$

$$179 = (4 + \sqrt{2}) \times 45 \rightarrow \frac{r\sqrt{2}}{\sin 45^\circ} = \frac{AC}{\sin 45^\circ} \rightarrow AC = \frac{\sqrt{2}}{2} \times \frac{r\sqrt{2}}{\frac{\sqrt{2}}{2}} = r\sqrt{2}$$

$$\frac{x}{\sin 45^\circ} = \frac{r\sqrt{2}}{\sin 45^\circ} \rightarrow x = r\sqrt{2} \times \frac{\sqrt{2}}{2} = r \quad (2)$$

$$\frac{\sin \hat{B}}{b-r} = \frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b} \rightarrow b-r = b \rightarrow b-b-r=0 \rightarrow b = -1 \times \quad (3)$$

$$b = -1 \times \quad (3)$$

$$b^2 = a^2 + c^2 - 2ac \cos B \rightarrow 179 = 4 + 2 - 2(2)(\sqrt{2}) \times \cos B \rightarrow \cos B = \frac{1}{\sqrt{2}} \rightarrow \sin B = \frac{1}{\sqrt{2}} \quad (4)$$

$$a^2 = b^2 + c^2 - 2bc \cos A \rightarrow a^2 = 17 + 17 + 8\sqrt{2} - 2(4)(\sqrt{2}) \times \frac{1}{\sqrt{2}} = 28 \rightarrow a = 2\sqrt{7}$$

$$\sin B = \frac{\sin A}{a} \rightarrow \sin B = \frac{\sqrt{2}}{2\sqrt{7}} \rightarrow \hat{B} = 45^\circ, \hat{C} = 90^\circ \quad (5)$$

$$BC^2 = AC^2 + AB^2 - 2(AC)(AB) \cos A \rightarrow 4 = 1 + 2 - 2(2)(1) \times \cos A \rightarrow \cos A = \frac{1}{2}$$

$$DE^2 = 17 + 17 - 2(4)(\sqrt{2}) \times \frac{1}{\sqrt{2}} \rightarrow DE = \sqrt{2} = 2\sqrt{1} \quad (6)$$

$$BC^2 = 4 + 2 - 2(2)(1) \times \frac{1}{2} \rightarrow BC = 2$$

$$S_{ABC} = \frac{1}{2} \times 2 \times 2 \times \sin 45^\circ = 1 \quad (7)$$

$$\frac{b^2 + c^2 + 2bc - a^2}{bc(\cos A + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos A)}{bc(\cos A + 1)} = \frac{2bc(1 + \cos A)}{bc(1 + \cos A)} = 2 \quad (8)$$

$$\frac{a^2 + b^2 - c^2}{a+b-c} = a \rightarrow a^2 + b^2 - c^2 = a^2 + ab - ac \rightarrow b^2 - c^2 = a(b-c)$$

$$\rightarrow (b-c)(b+c) = a(b-c) \xrightarrow{b \neq c} b+c = a \rightarrow a = b+c - 2bc \cos A$$

$$\rightarrow bc = -2bc \cos A \rightarrow \cos A = -\frac{1}{2} \quad (9)$$

$$\frac{1}{2}(3 - \cos \theta)(1 + \cos \theta) \times \frac{1}{2} = 2 \rightarrow 3 \cos \theta - \cos \theta + 1 = 4 \rightarrow \cos \theta = 1$$

$$\cos \theta = 1 \rightarrow b = 1 + 2 = 3, c = 3 - 1 = 2$$

$$a^2 = c^2 + b^2 - 2bc \cos A = 4 + 9 - 2(2)(3) \times \frac{\sqrt{2}}{2} = 1 \rightarrow a = 1 \quad (10)$$