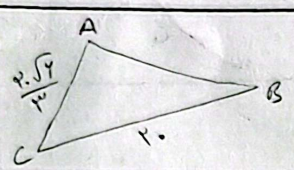
مبینا جعفری پور — تکلیف ۱۳ — دوازدهم دختر B

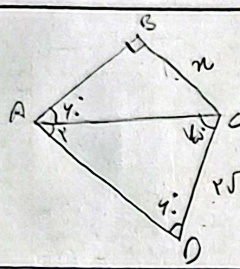
۱



$\hat{B}+\hat{C}=120° \Rightarrow \hat{A}=60° \Rightarrow \sin\hat{A}=\frac{\sqrt{3}}{2}$

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \Rightarrow \frac{20}{\frac{\sqrt3}{2}}=\frac{\frac{20\sqrt6}{3}}{\sin B} \Rightarrow \sin\hat B=\frac{\sqrt2}{2} \Rightarrow \boxed{\hat B=45°}$ یا $135°$ غ ق ق ($\hat B+\hat C>120°$) $\Rightarrow \boxed{\hat C=75°}$

۲

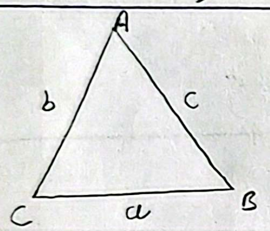


$\hat A_2=180°-(75°+60°)=45°$

$\sin 60°=\frac{x}{AC}=\frac{\sqrt3}{2} \Rightarrow AC=\frac{2x}{\sqrt3}$

قضیه سینوس‌ها در $\triangle ADC$: $\frac{CD}{\sin A_2}=\frac{AC}{\sin 60°} \Rightarrow \frac{2\sqrt2}{\frac{\sqrt2}{2}}=\frac{\frac{2x}{\sqrt3}}{\frac{\sqrt3}{2}} \Rightarrow \boxed{x=3}$

۳



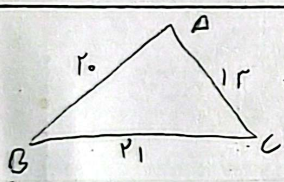
$(b^2-2)\frac{\sin\hat C}{\sin\hat B}=c \Rightarrow (b^2-2)\sin\hat C=c\sin\hat B = b\sin\hat C \xrightarrow{\sin\hat C\neq 0} b^2-2=b \Rightarrow b^2-b-2=0=(b-2)(b+1)$

قضیه سینوس‌ها: $\frac{b}{\sin\hat B}=\frac{c}{\sin\hat C} \Rightarrow b\sin\hat C=c\sin\hat B$

باید $\hat C$ صفر یا ۱۸۰ باشد که امکان‌پذیر نیست

$\Rightarrow \begin{cases} b=-1 & \text{غ ق ق} \\ \boxed{b=2} \end{cases}$

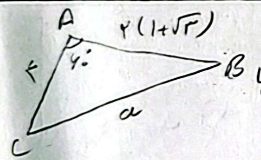
۴



قضیه کسینوس‌ها: $b^2=c^2+a^2-2ac\cdot\cos\hat B=(20)^2+(21)^2-2(20)(21)\cos\hat B=(13)^2 \Rightarrow \cos\hat B=\frac{8}{10} \Rightarrow \sin^2\hat B=1-\cos^2\hat B$

$=1-(\frac{8}{10})^2 \Rightarrow \sin\hat B=\pm\frac{6}{10} \xrightarrow[\sin\hat B>0]{\hat B<180°} \boxed{\sin\hat B=\frac{6}{10}}$

۵

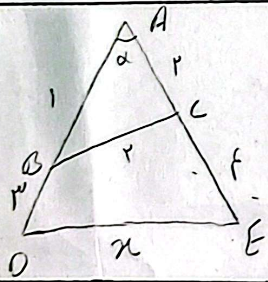


$a=\sqrt{b^2+c^2-2bc\cdot\cos60°}=\sqrt{16+16+8\sqrt3-8(1+\sqrt3)}=\sqrt{24}=\boxed{2\sqrt6}$

قضیه سینوس‌ها: $\frac{a}{\sin A}=\frac{b}{\sin\hat B}=\frac{c}{\sin\hat C} \Rightarrow \frac{2\sqrt6}{\frac{\sqrt3}{2}}=\frac{4}{\sin\hat B}=\frac{2(1+\sqrt3)}{\sin\hat C} \Rightarrow \sin\hat B=\frac{\sqrt2}{2} \Rightarrow$

- $\boxed{\hat B=45°} \Rightarrow \hat C=180°-(45°+60°)=\boxed{75°}$
- $\hat B=135°$ غ ق ق — $\hat A+\hat B>180°$ غیرممکن

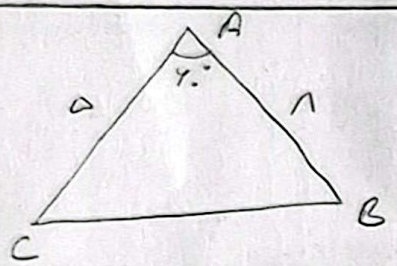
۶



$a^2=b^2+c^2-2bc\cdot\cos\alpha=4=4+1-4\cos\alpha \Rightarrow \cos\alpha=\frac14$

$x^2=d^2+e^2-2de\cdot\cos\alpha=36+16-48\times\frac14=40 \Rightarrow \boxed{x=2\sqrt{10}}$

۷



$a^2 = b^2 + c^2 - 2bc.\cos 60^\circ = 25 + 64 - 2(8)(5)\left(\frac{1}{2}\right) = 49 \Rightarrow \boxed{a = 7}$

$S_{ABC} = \frac{1}{2} \times b \times c \times \sin 60^\circ = \frac{1}{2} \times 5 \times 8 \times \frac{\sqrt{3}}{2} = \boxed{10\sqrt{3}}$

۸

$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc-a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc-(b^2+c^2-2bc.\cos\hat{A})}{bc(\cos\hat{A}+1)} = \frac{2bc(1+\cos\hat{A})}{bc(\cos\hat{A}+1)} = \boxed{2}$$

قضیه کسینوس‌ها

۹

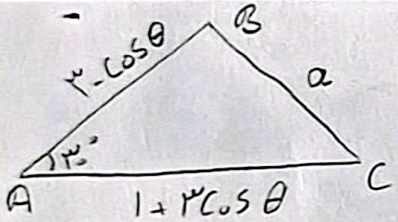
$$\frac{a^3+b^3-c^3}{a+b-c} = a^2 \Rightarrow a^3+b^3-c^3 = a^3+a^2b-a^2c \Rightarrow b^3-c^3 = a^2b-a^2c \Rightarrow (b-c)(b^2+c^2+bc) = a^2(b-c)$$

چون مثلث متمایز است پس $b \neq c$ و $b-c \neq 0$

$$\Rightarrow b^2+c^2+bc = a^2 = b^2+c^2-2bc.\cos\hat{A} \Rightarrow bc = -2bc\cos\hat{A} \Rightarrow \cos\hat{A} = \frac{-1}{2} \Rightarrow \boxed{\hat{A} = 120^\circ}$$

قضیه کسینوس‌ها

۱۰



$$S_{ABC} = \frac{1}{2} \times c \times b \times \sin 30^\circ = \frac{1}{2} \times (2-\cos\theta)(1+3\cos\theta) \times \frac{1}{2} = \frac{-3\cos^2\theta + 8\cos\theta + 3}{4} = 2 \Rightarrow -3\cos^2\theta + 8\cos\theta + 3 = 8$$

$$\Rightarrow 3\cos^2\theta - 8\cos\theta + 5 = 0 = (3\cos\theta - 5)(\cos\theta - 1) \Rightarrow \begin{cases} \cos\theta = \frac{5}{3} \text{ ق ق} \\ \cos\theta = 1 \longrightarrow b=4, c=2 \end{cases}$$

$a^2 = b^2+c^2-2bc\cos 30^\circ = 16+4-2(4)(2)\left(\frac{\sqrt{3}}{2}\right) = \boxed{20-8\sqrt{3}}$