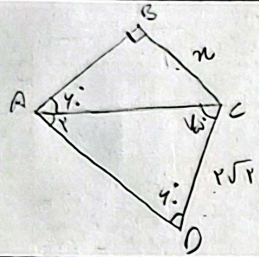


$$\hat{B} + \hat{C} = 120^\circ \Rightarrow \hat{A} = 60^\circ \Rightarrow \sin \hat{A} = \frac{\sqrt{3}}{2}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{2}{\frac{\sqrt{3}}{2}} = \frac{2}{\sin B} \Rightarrow \sin B = \frac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60^\circ \text{ یا } 120^\circ$$

غیر ممکن

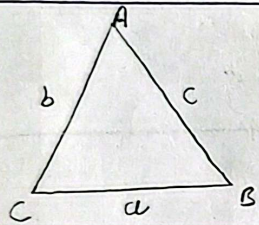
$$\hat{C} = 120^\circ \Rightarrow \hat{C} = 120^\circ$$



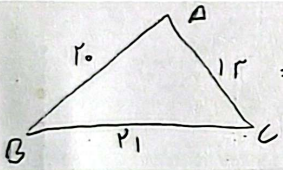
$$\hat{A}_r = 180^\circ - (120^\circ + 60^\circ) = 0^\circ$$

$$\sin 60^\circ = \frac{AD}{AC} = \frac{\sqrt{3}}{2} \Rightarrow AC = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

$$\text{تعیین سینوس خارج AD: } \frac{CD}{\sin A_r} = \frac{AC}{\sin 60^\circ} \Rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{2}{\frac{\sqrt{3}}{2}} \Rightarrow x = 2$$

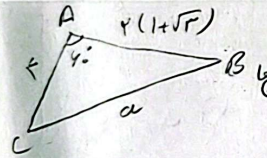


$$(b^2 - 1) \frac{\sin \hat{C}}{\sin \hat{B}} = C \Rightarrow (b^2 - 1) \sin \hat{C} = C \sin \hat{B} \Rightarrow b \sin \hat{C} = C \sin \hat{B} \Rightarrow b^2 - 1 = b \Rightarrow b^2 - b - 1 = 0 \Rightarrow (b-2)(b+1) = 0 \Rightarrow b = 2$$



$$b^2 = c^2 + a^2 - 2ac \cos \hat{B} = (20)^2 + (21)^2 - 2(20)(21) \cos \hat{B} = (13)^2 \Rightarrow \cos \hat{B} = \frac{1}{10} \Rightarrow \sin \hat{B} = 1 - \cos \hat{B} = \frac{9}{10}$$

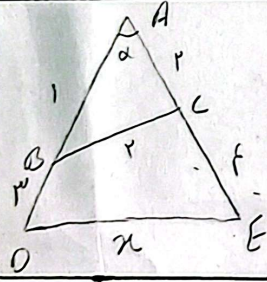
$$\sin \hat{B} = \frac{4}{10} \Rightarrow \sin \hat{B} = \frac{2}{5}$$



$$a = \sqrt{b^2 + c^2 - 2bc \cos \hat{A}} = \sqrt{14 + 14 - 2(14)(14) \cos \hat{A}} = \sqrt{24} = 2\sqrt{6}$$

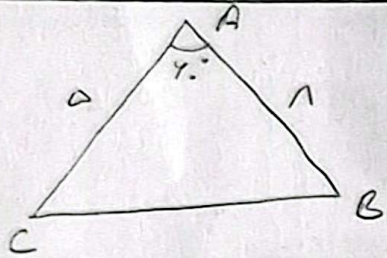
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{2\sqrt{6}}{\frac{\sqrt{3}}{2}} = \frac{2}{\sin B} = \frac{2(1+\sqrt{3})}{\sin C} \Rightarrow \sin B = \frac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60^\circ \text{ یا } 120^\circ$$

غیر ممکن



$$a^2 = b^2 + c^2 - 2bc \cos \alpha = 4 = 4 + 4 - 2 \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2}$$

$$x^2 = d^2 + e^2 - 2de \cos \alpha = 34 + 14 - 2 \times \frac{1}{2} = 40 \Rightarrow x = 2\sqrt{10}$$



$$a^2 = b^2 + c^2 - 2bc \cos 40^\circ = 2^2 + 2^2 - 2(2)(2) \left(\frac{1}{\sqrt{2}}\right) = 4 \Rightarrow \boxed{a = \sqrt{2}}$$

$$S_{ABC} = \frac{1}{2} \times b \times c \times \sin 40^\circ = \frac{1}{2} \times 2 \times 2 \times \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2}}$$

✓

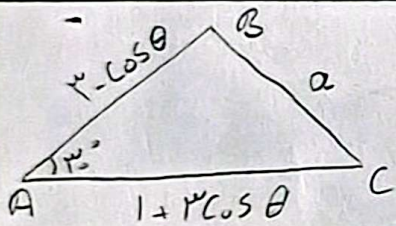
$$\frac{(b+c)^2 - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - a^2}{bc(\cos \hat{A} + 1)} = \frac{b^2 + c^2 + 2bc - (b^2 + c^2 - 2bc \cos \hat{A})}{bc(\cos \hat{A} + 1)} = \frac{2bc(1 + \cos \hat{A})}{bc(\cos \hat{A} + 1)} = \boxed{2}$$

✓

$$\frac{a^2 + b^2 - c^2}{a + b - c} = a^2 \Rightarrow a^2 + b^2 - c^2 = a^2 + a^2 b - a^2 c \Rightarrow b^2 - c^2 = a^2 b - a^2 c \Rightarrow (b-c)(b+c) = a^2(b-c) \quad b \neq c \Rightarrow b+c = a^2$$

$$\Rightarrow b^2 + c^2 + bc = a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow bc = -2bc \cos \hat{A} \Rightarrow \cos \hat{A} = -\frac{1}{2} \Rightarrow \boxed{A = 120^\circ}$$

✓



$$S_{ABC} = \frac{1}{2} \times b \times c \times \sin 120^\circ = \frac{1}{2} \times (2 - \cos \theta) \times (1 + 3 \cos \theta) \times \frac{\sqrt{3}}{2} = 2 \Rightarrow -3 \cos^2 \theta + 11 \cos \theta + 3 = 0$$

$$\Rightarrow 3 \cos^2 \theta - 11 \cos \theta + 3 = 0 \Rightarrow (3 \cos \theta - 1)(\cos \theta - 3) = 0 \Rightarrow \begin{cases} \cos \theta = \frac{1}{3} \\ \cos \theta = 3 \end{cases}$$

$$a^2 = b^2 + c^2 - 2bc \cos 120^\circ = 1^2 + 3^2 - 2(1)(3) \left(-\frac{1}{2}\right) = 10 \Rightarrow \boxed{a = \sqrt{10}}$$

✓