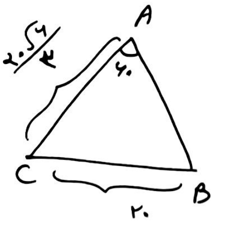
تکلیف شماره ۱۳ نیکا بابایی

-۱

$\overset{\triangle}{ABC}$ در $\longrightarrow \hat{B}+\hat{C} = 120 \longrightarrow \hat{A} = 60$



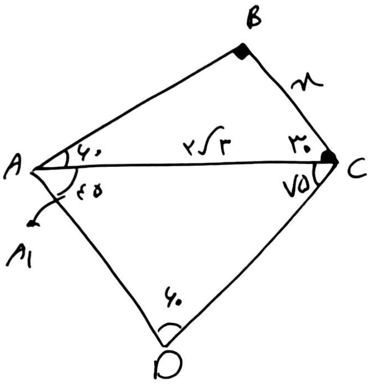
$$\frac{a}{\sin 60} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{20}{\frac{\sqrt3}{2}} = \frac{\frac{20\sqrt6}{3}}{\sin B} \longrightarrow \sin B = \frac{\frac{\sqrt3}{2}\times\frac{20\sqrt6}{3}}{20} = \frac{\sqrt{18}}{6} = \frac{\sqrt3}{2}$$

$$\hat{B} = \frac{\pi}{4} = 45^\circ \qquad C = 120 - 45 = 75^\circ \qquad \boxed{B = 45^\circ}$$

$$\boxed{C = 75^\circ}$$

-۲



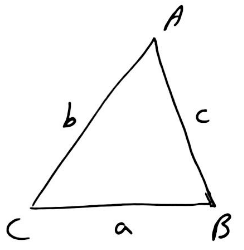
$$A_1 = 180 - (60+75) = 45$$

$$\frac{CD}{\sin \hat{A_1}} = \frac{AC}{\sin \hat{D}} \longrightarrow \frac{2\sqrt2}{\sin 45} = \frac{AC}{\sin 60}$$

$$\frac{2\sqrt2}{\frac{\sqrt2}{2}} = \frac{AC}{\frac{\sqrt3}{2}} \longrightarrow \boxed{AC = 2\sqrt3}$$

$\overset{\triangle}{ABC}$ در $\longrightarrow \frac{x}{\sin 60} = \frac{2\sqrt3}{\sin 90} \longrightarrow \frac{x}{\frac{\sqrt3}{2}} = \frac{2\sqrt3}{1} \longrightarrow \boxed{x = 3}$

-۳



$$(b^2-2)\left(\frac{\sin\hat{C}}{\sin B}\right) = c$$

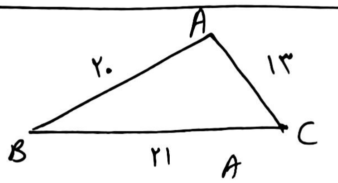
$$\frac{c}{\sin\hat{C}} = \frac{b}{\sin\hat{B}} = \frac{a}{\sin\hat{A}} = K \qquad \begin{cases} \sin\hat{C} = \frac{c}{K} \\ \sin\hat{B} = \frac{b}{K} \\ \sin\hat{A} = \frac{a}{K} \end{cases}$$

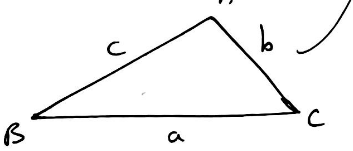
$$\longrightarrow \frac{\sin\hat{C}}{\sin\hat{B}} = \frac{\frac{c}{\cancel{K}}}{\frac{b}{\cancel{K}}} = \frac{c}{b} \longrightarrow (b^2-2)\left(\frac{\cancel{c}}{b}\right) = \cancel{c} \longrightarrow \frac{b^2-2}{b} = 1$$

$$\longrightarrow b^2-2 = b \longrightarrow b^2-b-2=0 \longrightarrow (b-2)(b+1) = 0 \longrightarrow \boxed{b = 2}$$ ✓قق

$\hookrightarrow b = -1$ غقق

-۴



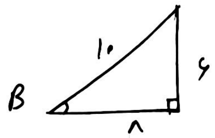


$$b^2 = a^2 + c^2 - 2ac\cos\hat{B}$$

$$13^2 = 20^2 + 21^2 - 2(20)(21)\cos\hat{B}$$

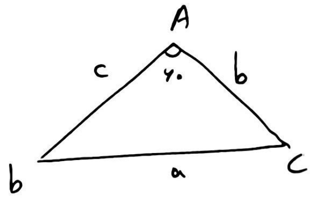
$$2(20)(21)\cos B = 20^2 + 21^2 - 13^2$$

$$\cos\hat{B} = 0.8 \longrightarrow \boxed{\sin\hat{B} = 0.6}$$ جواب



نسکاب

۵ - الف)



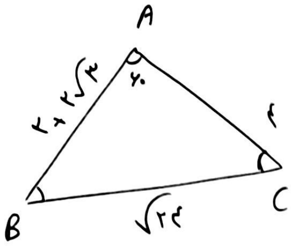
$b = ۴ \qquad c = ۲(۱+\sqrt{۳}) \qquad \hat{A} = ۶۰°$

$a^۲ = b^۲ + c^۲ - ۲bc \cos ۶۰$

$a^۲ = ۱۶ + (۲+۲\sqrt{۳})^۲ - \cancel{۲}(۴)(۲+۲\sqrt{۳}) \times \frac{۱}{\cancel{۲}}$

$a^۲ = ۱۶ + ۱۶ + \cancel{۸\sqrt{۳}} - ۸ - \cancel{۸\sqrt{۳}} = ۲۴ \longrightarrow \boxed{a = \sqrt{۲۴}}$

ب)



$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

$$\frac{\sqrt{۲۴}}{\frac{\sqrt{۳}}{۲}} = \frac{۴}{\sin B} = \frac{۲+۲\sqrt{۳}}{\sin \hat{C}}$$

$\sin \hat{B} = \frac{۴ \times \frac{\sqrt{۳}}{۲}}{\sqrt{۲۴}} = \frac{\sqrt{۲}}{۲} \longrightarrow \boxed{B = ۴۵°}$

$\hat{C} = ۱۸۰ - (۶۰ + ۴۵) = ۷۵ \longrightarrow \boxed{C = ۷۵°}$

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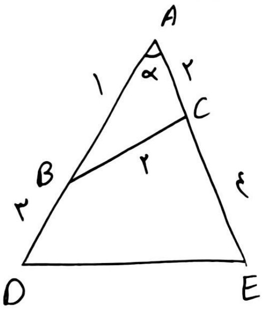
۶ -



$BC^۲ = AB^۲ + AC^۲ - ۲(AB)(AC) \cos \alpha$

$\cancel{۴} = ۱ + \cancel{۴} - ۲(۱)(۲) \cos \alpha$

$۴ \cos \alpha = ۱ \longrightarrow \cos \alpha = \frac{۱}{۴}$

$DE^۲ = AD^۲ + AE^۲ - ۲(AD)(AE) \cos \hat{A}$

$DE^۲ = (۴)^۲ + (۶)^۲ - ۲(\cancel{۴})(۶)(\frac{۱}{\cancel{۴}}) \longrightarrow DE^۲ = ۱۶ + ۳۶ - ۱۲ = ۴۰$

$\boxed{DE = \sqrt{۴۰}}$ جواب

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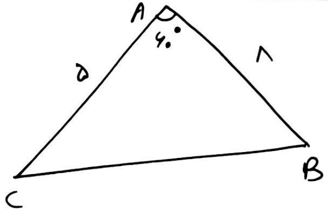
۷ -

الف)



$BC^۲ = AC^۲ + AB^۲ - ۲(AC)(AB) \cos ۶۰$

$BC^۲ = ۲۵ + ۶۴ - \cancel{۲}(۵)(۸) \times \frac{۱}{\cancel{۲}}$

$BC^۲ = ۴۹ \longrightarrow \boxed{BC = ۷}$ جواب

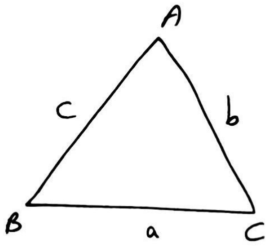
ب)

$S_{ABC}^{\Delta} = \frac{۱}{۲} AB \times AC \times \sin \hat{A}$

$S_{ABC}^{\Delta} = \frac{۱}{۲} \times ۵ \times ۸ \times \frac{\sqrt{۳}}{۲} = \boxed{۱۰\sqrt{۳}}$

جواب

نیک‌بابت



۸ -

$$\frac{(b+c)^۲-a^۲}{bc(\cos\hat{A}+۱)}=?$$

$$a^۲=b^۲+c^۲-۲bc\cos\hat{A}$$

$$(b+c)^۲=b^۲+c^۲+۲bc$$

$$\longrightarrow \frac{(b+c)^۲-a^۲}{bc(\cos\hat{A}+۱)}=\frac{\cancel{b^۲}+\cancel{c^۲}+۲bc-\cancel{b^۲}-\cancel{c^۲}+۲bc\cos\hat{A}}{bc(\cos\hat{A}+۱)}=\frac{۲bc+۲bc\cos\hat{A}}{bc(\cos\hat{A}+۱)}$$

$$=\frac{۲\cancel{bc}\,\cancel{(۱+\cos\hat{A})}}{\cancel{bc}\,\cancel{(\cos\hat{A}+۱)}}=\boxed{۲}$$ جواب

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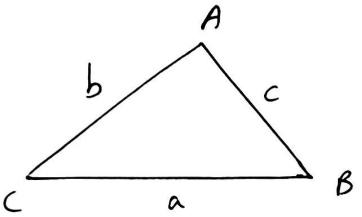
۹ -



$$\frac{a^۳+b^۳-c^۳}{a+b-c}=a^۲$$

$$\cancel{a^۳}+b^۳-c^۳=\cancel{a^۳}+a^۲b-a^۲c \longrightarrow b^۳-a^۲b=c^۳-a^۲c$$

$$\longrightarrow b(b^۲-a^۲)=c(c^۲-a^۲) \longrightarrow b(\cancel{b^۲}-\cancel{b^۲}-c^۲+۲bc\cos A)=c(\cancel{c^۲}-b^۲-\cancel{c^۲}+۲bc\cos A)$$

$$\longrightarrow b(۲bc\cos\hat{A}-c^۲)=c(۲bc\cos\hat{A}-b^۲)$$

$$\cancel{bc}(۲b\cos\hat{A}-c)=\cancel{bc}(۲c\cos\hat{A}-b)$$

$$\longrightarrow ۲b\cos\hat{A}-c=۲c\cos\hat{A}-b \longrightarrow ۲b\cos\hat{A}-۲c\cos\hat{A}=c-b$$

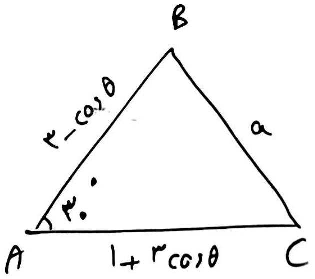
$$۲\cos\hat{A}(\cancel{b-c})=\cancel{c-b}$$

$$۲\cos\hat{A}=-۱ \longrightarrow \boxed{\cos\hat{A}=\frac{-۱}{۲}}$$

جواب

[illegible]

۱۰ _



$S_{\overset{\Delta}{ABC}} = ۲$

$S_{\overset{\Delta}{ABC}} = \frac{۱}{۲}(۳-\cos\theta)(۱+۳\cos\theta)\underbrace{\sin ۳۰}_{\frac{۱}{۲}}$

$\frac{۱}{۴}(۳-\cos\theta)(۱+۳\cos\theta) = ۲ \longrightarrow (۱+۳\cos\theta)(۳-\cos\theta) = ۸$

$۳+۹\cos\theta-\cos\theta-۳\cos^۲\theta = ۸$

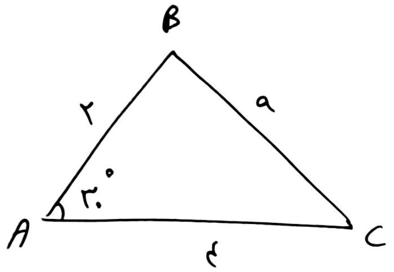
$۳\cos^۲\theta-۸\cos\theta+۵ = ۰$

جمع ضرایب صفر ← یکی از ریشه‌ها ۱

← دیگری $\frac{c}{a}$

$\boxed{\cos\theta = ۱}$ ✓ ق ق

$\cos\theta = \frac{۵}{۳}$ × غ ق ق



$a^۲ = b^۲+c^۲-۲bc\cos\hat{A}$

$a^۲ = ۱۶+۴-۲(۴)(۲)\underbrace{\cos ۳۰}_{\frac{\sqrt{۳}}{۲}}$

جواب $\boxed{a^۲ = ۲۰-۸\sqrt{۳}}$