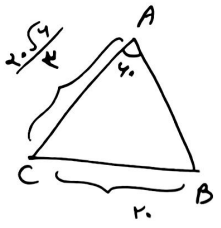
تکلیف شماره ۱۳

۲۰ آذر

۱-

$\hat{A}+\hat{B}+\hat{C}=180^\circ \longrightarrow \hat{B}+\hat{C}=120^\circ \longrightarrow \hat{A}=60^\circ$



$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

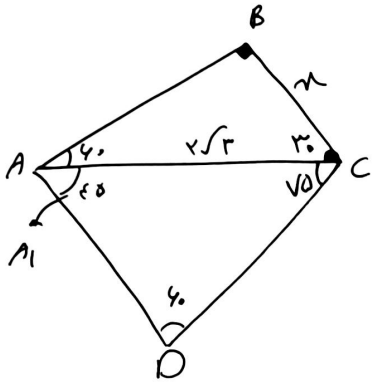
$$\frac{20}{\frac{\sqrt{3}}{2}}=\frac{\frac{20\sqrt{6}}{3}}{\sin B} \longrightarrow \sin B=\frac{\frac{\sqrt{3}}{2}\times\frac{20\sqrt{6}}{3}}{20}=\frac{\sqrt{18}}{6}=\frac{\sqrt{3}}{2}$$

$\hat{B}=\frac{\pi}{3}=60^\circ$ $\quad C=120-60=75^\circ$

$B=60^\circ$

$C=75^\circ$

۲-



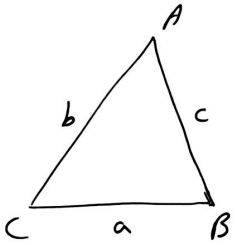
$A_1=180-(60+75)=45$

$$\frac{CD}{\sin A_1}=\frac{AC}{\sin D} \longrightarrow \frac{2\sqrt{2}}{\sin 45}=\frac{AC}{\sin 60}$$

$$\frac{2\sqrt{2}}{\frac{\sqrt{2}}{2}}=\frac{AC}{\frac{\sqrt{3}}{2}} \longrightarrow AC=2\sqrt{3}$$

$\triangle ABC \sim \longrightarrow \frac{x}{\sin 60}=\frac{2\sqrt{3}}{\sin 90} \longrightarrow \frac{x}{\frac{\sqrt{3}}{2}}=\frac{2\sqrt{3}}{1} \longrightarrow x=3$

۳-



$$(b^2-2)\left(\frac{\sin C}{\sin B}\right)=c$$

$$\frac{c}{\sin C}=\frac{b}{\sin B}=\frac{a}{\sin A}=k \qquad \begin{cases}\sin C=\frac{c}{k}\\ \sin B=\frac{b}{k}\\ \sin A=\frac{a}{k}\end{cases}$$

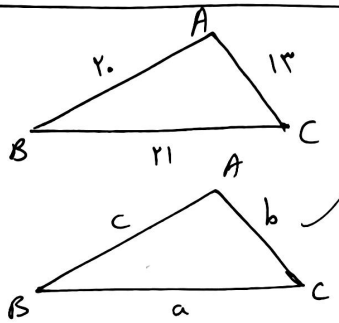
$$\longrightarrow \frac{\sin C}{\sin B}=\frac{\frac{c}{k}}{\frac{b}{k}}=\frac{c}{b} \longrightarrow (b^2-2)\left(\frac{c}{b}\right)=c \longrightarrow \frac{b^2-2}{b}=1$$

$$\longrightarrow b^2-2=b \longrightarrow b^2-b-2=0 \longrightarrow (b-2)(b+1)=0$$

قبول ✓ $b=2$

غ ق ق ✗ $b=-1$

۴-



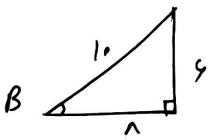
$$b^2=a^2+c^2-2ac\cos B$$

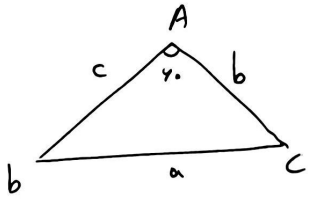
$$13^2=20^2+21^2-2(20)(21)\cos B$$

$$2(20)(21)\cos B=20^2+21^2-13^2$$

$$\cos B=0.8 \longrightarrow \sin B=0.6$$

جواب





$b = ۴ \qquad c = ۲(۱+\sqrt{۳}) \qquad \hat{A} = ۶۰^\circ$

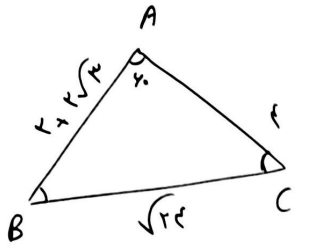
تمرینات

۵ - الف)

$a^۲ = b^۲ + c^۲ - ۲bc\cos ۶۰$

$a^۲ = ۱۶ + (۲+۲\sqrt{۳})^۲ - ۴(۴)(۲+۲\sqrt{۳}) \times \frac{۱}{۲}$

$a^۲ = ۱۶ + ۱۶ + ۸\sqrt{۳} - ۸\sqrt{۳} = ۲۴ \longrightarrow a = \sqrt{۲۴}$



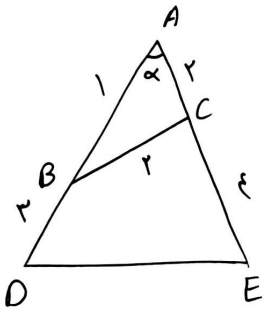
ب)

$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}}$

$\frac{\sqrt{۲۴}}{\frac{\sqrt{۳}}{۲}} = \frac{۴}{\sin B} = \frac{۲+۲\sqrt{۳}}{\sin\hat{C}}$

$\sin\hat{B} = \frac{۴ \times \frac{\sqrt{۳}}{۲}}{\sqrt{۲۴}} = \frac{\sqrt{۲}}{۲} \longrightarrow B = ۴۵^\circ$

$\hat{C} = ۱۸۰ - (۶۰ + ۴۵) = ۷۵ \longrightarrow C = ۷۵^\circ$



۶ -

$BC^۲ = AB^۲ + AC^۲ - ۲(AB)(AC)\cos\alpha$

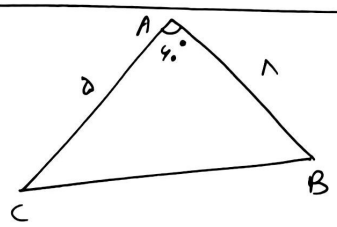
$۴ = ۱ + ۴ - ۲(۱)(۲)\cos\alpha$

$۴\cos\alpha = ۱ \longrightarrow \cos\alpha = \frac{۱}{۴}$

$DE^۲ = AD^۲ + AE^۲ - ۲(AD)(AE)\cos\hat{A}$

$DE^۲ = (۴)^۲ + (۶)^۲ - ۲(۴)(۶)\left(\frac{۱}{۴}\right) \longrightarrow DE^۲ = ۱۶ + ۳۶ - ۱۲ = ۴۰$

جواب $DE = \sqrt{۴۰}$



۷ -

الف)

$BC^۲ = AC^۲ + AB^۲ - ۲(AC)(AB)\cos ۶۰$

$BC^۲ = ۲۵ + ۶۴ - ۲(۵)(۸) \times \frac{۱}{۲}$

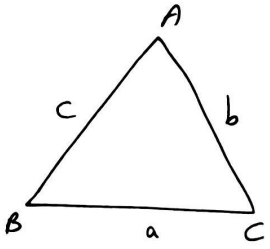
$BC^۲ = ۴۹ \longrightarrow BC = ۷$ جواب

ب)

$S_{ABC}^{\Delta} = \frac{۱}{۲} AB \times AC \times \sin\hat{A}$

$S_{ABC}^{\Delta} = \frac{۱}{۲} \times ۵ \times ۸ \times \frac{\sqrt{۳}}{۲} = ۱۰\sqrt{۳}$

جواب



$$\frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = ?$$

هندسه

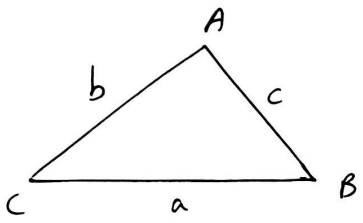
۸-

$$a^2 = b^2 + c^2 - 2bc\cos\hat{A}$$

$$(b+c)^2 = b^2 + c^2 + 2bc$$

$$\longrightarrow \frac{(b+c)^2 - a^2}{bc(\cos\hat{A}+1)} = \frac{b^2+c^2+2bc - b^2 - c^2 + 2bc\cos\hat{A}}{bc(\cos\hat{A}+1)} = \frac{2bc + 2bc\cos\hat{A}}{bc(\cos\hat{A}+1)}$$

$$= \frac{2bc(1+\cos\hat{A})}{bc(\cos\hat{A}+1)} = 2$$ جواب



$$\frac{a^3 + b^3 - c^3}{a+b-c} = a^2$$

۹-

$$a^3 + b^3 - c^3 = a^3 + a^2b - a^2c \longrightarrow b^3 - a^2b = c^3 - a^2c$$

$$\longrightarrow b(b^2 - a^2) = c(c^2 - a^2) \longrightarrow b(b^2 - b^2 - c^2 + 2bc\cos A) = c(c^2 - b^2 - c^2 + 2bc\cos A)$$

$$\longrightarrow b(2bc\cos\hat{A} - c^2) = c(2bc\cos\hat{A} - b^2)$$

$$bc(2b\cos\hat{A} - c) = bc(2c\cos\hat{A} - b)$$

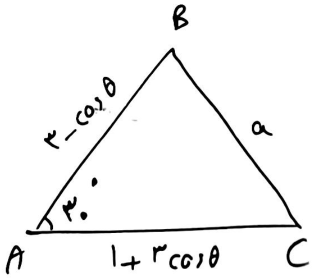
$$\longrightarrow 2b\cos\hat{A} - c = 2c\cos\hat{A} - b \longrightarrow 2b\cos\hat{A} - 2c\cos\hat{A} = c - b$$

$$2\cos\hat{A}(b-c) = c-b$$

$$2\cos\hat{A} = -1 \longrightarrow \cos\hat{A} = \frac{-1}{2}$$

جواب

۱۰ _



$S_{\overset{\Delta}{ABC}} = ۲$

$S_{\overset{\Delta}{ABC}} = \frac{1}{۲}(۳-\cos\theta)(1+۳\cos\theta)\underbrace{\sin ۳۰}_{\frac{1}{۲}}$ ۲

$\frac{1}{۴}(۳-\cos\theta)(1+۳\cos\theta) = ۲ \longrightarrow (1+۳\cos\theta)(۳-\cos\theta) = ۸$

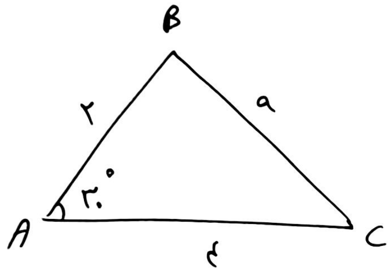
$۳ + ۹\cos\theta - \cos\theta - ۳\cos^۲\theta = ۸$

$۳\cos^۲\theta - ۸\cos\theta + ۵ = ۰$

جمع ضرایب صفر ← یکی از ریشه‌ها ۱ و ریشه دیگر $\frac{c}{a}$

$\cos\theta = 1$ ✓ ق ق

$\cos\theta = \frac{۵}{۳}$ X غ ق ق



$a^۲ = b^۲ + c^۲ - ۲bc\cos\hat{A}$

$a^۲ = ۱۶ + ۴ - ۲(۴)(۲)\underbrace{\cos ۳۰}_{\frac{\sqrt{۳}}{۲}}$

جواب $a^۲ = ۲۰ - ۸\sqrt{۳}$