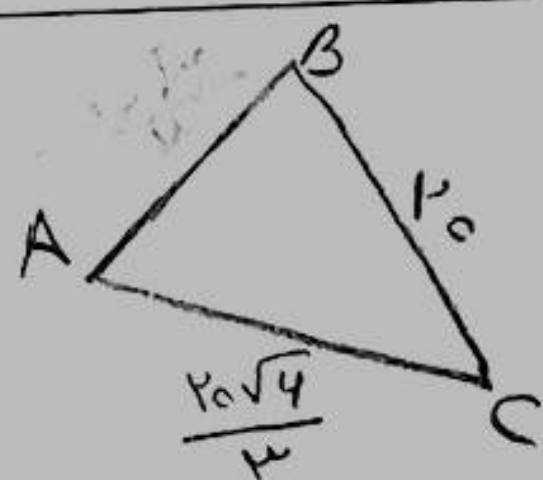




$$\hat{B} + \hat{C} = 120^\circ \Rightarrow \hat{B} = 120^\circ - \hat{C} \Rightarrow A = 40^\circ$$

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} \Rightarrow 10 \sin B = \frac{10\sqrt{4}}{3} \sin 40^\circ \Rightarrow \sin B = \frac{\sqrt{18}}{4} = \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{2} \Rightarrow \boxed{B = 45^\circ} \text{ و } \boxed{C = 75^\circ}$$

۱

$$A' = 40^\circ$$

$$\frac{CP}{\sin 60^\circ} = \frac{AC}{\sin 40^\circ} \Rightarrow \frac{10\sqrt{3}}{2} = \frac{AC}{\frac{\sqrt{3}}{2}} \Rightarrow AC = 10\sqrt{3}$$

$$\frac{AC}{\sin B_{90}} = \frac{x}{\sin 40^\circ} \Rightarrow \frac{10\sqrt{3}}{1} = \frac{x}{\frac{\sqrt{3}}{2}} \Rightarrow \boxed{x = 3}$$

۲

$$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \frac{\sin C}{\sin B} = \frac{c}{b} = \frac{c}{b^2 - 1} \Rightarrow b^2 - 1 = b \Rightarrow b^2 - b - 1 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0 \Rightarrow \begin{cases} b = -1 \times \\ b = 2\sqrt{2} \end{cases}$$

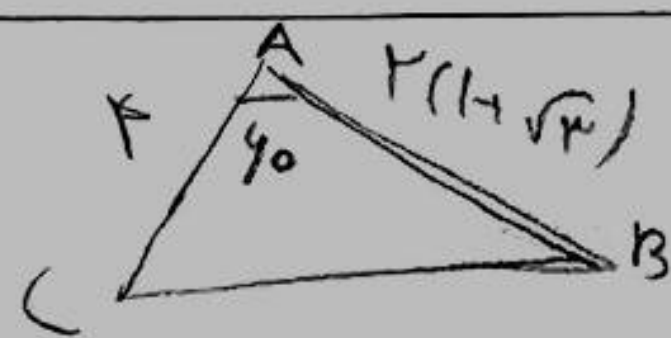
۳

$$a^2 = b^2 + c^2 - 2 \cdot AB \cdot BC \cdot \cos B \Rightarrow 13^2 = 11^2 + 10^2 - 2 \cdot 11 \cdot 10 \cdot \cos B \Rightarrow$$

$$169 = 121 + 100 - 220 \cos B \Rightarrow 220 \cos B = 52 \Rightarrow \cos B = \frac{13}{55} \Rightarrow \sin^2 B + \cos^2 B = 1$$

$$\Rightarrow \sin^2 B = 1 - \frac{169}{3025} = \frac{9}{3025} \Rightarrow \boxed{\sin B = \frac{3}{55}}$$

۴



$$a^2 = 10^2 + (10(1+\sqrt{3}))^2 - 2 \cdot 10 \cdot 10(1+\sqrt{3}) \cdot \cos 40^\circ$$

$$\Rightarrow a^2 = 100 + 100(1+\sqrt{3})^2 - 200(1+\sqrt{3}) \cdot \frac{\sqrt{3}}{2} \Rightarrow a^2 = 100 + 100(1+2\sqrt{3}+3) - 100(1+\sqrt{3})\sqrt{3}$$

$$= 100 + 400 + 200\sqrt{3} - 100\sqrt{3} - 100 = 100 + 300\sqrt{3}$$

$$\Rightarrow a = \sqrt{100 + 300\sqrt{3}}$$

۵

مثلاً
کریم

$$a^2 = c^2 + b^2 - 2bc \cos \alpha \Rightarrow r = 1 + r - r \cos \alpha \Rightarrow r \cos \alpha = 1 \Rightarrow \cos \alpha = \frac{1}{r}$$

مثلاً
نیر

$$a^2 = d^2 + e^2 - 2de \cos \alpha \Rightarrow a^2 = 3^2 + 1^2 - 2 \times 3 \times 1 \times \frac{1}{3} = 6 \Rightarrow a = \sqrt{6} = \boxed{2\sqrt{1.5}}$$

$$\frac{1}{r} b \cdot c \times \sin A = \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} \times \frac{1}{r} = \frac{1}{r^4} \quad | -$$

$$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \Rightarrow a^2 = \omega^2 + 1^2 - 2(\omega)(1)(\cos \hat{A}) \quad | -$$

$$= 19 - 2\omega = 1 \Rightarrow \omega = 9 \Rightarrow \boxed{a = 10}$$

$$(b+c)^2 = b^2 + 2bc + c^2 \Rightarrow (b+c)^2 - a^2 = b^2 + 2bc + c^2 - a^2, a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Rightarrow \cancel{b^2} + 2bc + \cancel{c^2} - \cancel{b^2} - \cancel{c^2} + 2bc \cos A = 2bc + 2bc \cos A = 2bc(1 + \cos A)$$

$$\Rightarrow \frac{(b+c)^2 - a^2}{bc(\cos A + 1)} = \frac{2bc(1 + \cos A)}{bc(\cos A + 1)} = \boxed{2}$$

$$a^2 = \frac{a^3 + b^3 - c^3}{a+b-c} \Rightarrow a^3 + a^2b - a^2c = a^3 + b^3 - c^3 \Rightarrow a^2b - a^2c = b^3 - c^3$$

$$\Rightarrow a^2(b-c) = b^3 - c^3 \Rightarrow a^2(\cancel{b-c}) = (\cancel{b-c})(b^2 + bc + c^2) \Rightarrow a^2 = b^2 + bc + c^2$$

$$b^2 + bc + c^2 = b^2 + c^2 - 2bc \cos A \Rightarrow \cancel{b^2} + \cancel{c^2} + bc = -2\cancel{b} \cancel{c} \cos A \Rightarrow \boxed{\cos A = -\frac{1}{2}}$$

$$\frac{1}{r} (r - \cos \theta)(1 + r \cos \theta) \times \frac{1}{r} = r \Rightarrow r + r^2 \cos \theta - \cos \theta - r^2 \cos^2 \theta = 1$$

$$\Rightarrow r^2 \cos^2 \theta - 1 \cos \theta + r = 0 \Rightarrow \cos \theta = \frac{1}{r} \quad | - \cos A = \frac{1}{r} \times$$

$$AB = r$$

$$AC = r \Rightarrow a^2 = r^2 + r^2 - 2 \times r \times r \times \cos 120^\circ = 14 + r - 14 \times \frac{1}{r} = \boxed{r^2 - 14/r}$$